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REASONING, LOGIC, AND CATEGORY MISTAKES

[RAZONAMIENTO, LÓGICA Y ERRORES CATEGORIALES]

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ABSTRACT: In this contribution we sketch a propositional logical system designed to represent reasoning with philosophical categories. This should be of relative interest, at least, for two reasons. In first place, the proposed system attempts to formalize the notion of category mistake; and, in second place, the system provides a formal alternative to regulate reasoning involving categories, since the propositional systems typically used to represent reasoning are unable to do that, thus allowing the introduction of category mistakes..
KEYWORDS: Logical system; category system; product logic; ontology.

RESUMO: En esta contribución bosquejamos un sistema lógico proposicional diseñado para representar razonamiento con categorías filosóficas. Esto debería ser de cierto interés, al menos, por dos razones. En primer lugar, el sistema propuesto pretende formalizar la noción de error categorial; y, en segundo lugar, el sistema provee una alternativa formal para regimentar razonamientos que incluyen categorías, ya que los sistemas proposicionales usados típicamente para representar razonamiento son incapaces de hacerlo, lo cual permite la introducción de errores categoriales.
PALAVRAS CLAVE: Sistema lógico; sistema de categorías; lógica producto; ontología.

> *4*.112 Der Zweck der Philosophie ist die logische Klärung der Gedanken. Die Philosophie ist keine Lehre, sondern eine Tätigkeit. Ein philosophisches Werk besteht wesentlich aus Erläuterungen. Das Resultat der Philosophie sind nicht "philosophische Sätze", sondern das Klarwerden von Sätzen. Die Philosophie soll die Gedanken, die sonst, gleichsam, trübe und verschwommen sind, klar machen und scharf abgrenzen.
>
> *Wittgenstein*

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1 Introduction

To give an idea of the scope of this contribution, we would like to begin with a particular re-interpretation of proposition 4.112 from Wittgenstein's *Tractatus*: the object of a logical system is the logical elucidation of inference. A logical system is not a doctrine but an activity. A logical work consists essentially of elucidations. The result of a logical system is not a number of "logical propositions", but to make propositions clear. A logical system should make clear and delimit sharply the inferences which otherwise are, as it were, opaque and blurred.[1]

In this work, we look forward the elucidation of inference; in particular, our main goal is to sketch a propositional logical system designed to represent reasoning with philosophical categories. We will explain what these concepts entail in due time, but meanwhile, we would like to mention that this goal should be of relative interest, at least, for two reasons. In first place, the proposed system attempts to formalize the notion of category mistake; and, in second place, the system provides a formal alternative to regulate reasoning involving categories, since the propositional systems typically used to represent reasoning are unable to do that, thus allowing the introduction of category mistakes or high complexity analyses, as we will see with some examples.

To reach our goal, we have organized the content in the next way. In Section 2, we briefly review the notions of logical system and category mistake. Then, in Section 3, we sketch a logical system by proposing some semantics, natural deduction rules, and logical properties. Finally, in Section 4, we close with a summary.

2 Reasoning, logical systems, and category mistakes

Reasoning is a process that produces new information given previous data, by following certain norms that allow us to describe inference as the unit of measurement of reasoning: inference may be more or less (in)correct depending on the compliance or violation of such norms. Logical systems, the tools used to model and better understand inference, may be defined by pairs of the form $<S,B>$, where S stands for a signature, and B for a semantic base (often equivalent to a calculus). Usually, some syntax is used to determine, uniquely and recursively, the well formed expressions of the system; while semantics is used to provide meaning to such expressions.

To illustrate this notion, let us consider classical propositional logic (L_p), which is a scalar system that has been typically used to represent reasoning at a propositional level. Its vocabulary includes constants, CONS=$\{\neg, \wedge\}$, and variables, VAR=$\{\varphi, \psi, \ldots\}$. Its syntax is defined by two rules: *i)* if $\varphi\in$VAR, then φ is a well formed formula (wff) of L_p; and *ii)* if φ and ψ are wffs of L_p, then $\neg\varphi$ and $\varphi\wedge\psi$ are also wffs of L_p. Finally, its semantics is composed by a domain of truth values, D=$\{1, 0\}$, where 1 stands for the designated value and 0 for the anti-designated value; and a function of interpretation that maps the variables to truth values, f:VAR™D, so that for all $\varphi\in$VAR, either $f(\varphi)=1$ or $f(\varphi)=0$, and for no $\varphi\in$VAR, $f(\varphi)=1$ and $f(\varphi)=0$. Given this function, a valuation v_{Lp} is defined in such way that $v_{Lp}(\varphi)=f(\varphi)$, $v_{Lp}(\neg\varphi)=1-v_{Lp}(\varphi)$, and $v_{Lp}(\varphi\wedge\psi)=\min(v_{Lp}(\varphi),v_{Lp}(\psi))$, thus defining negation and conjunction. This valuation

1 Wittgenstein, of course, was not talking about logical systems, but philosophy (Wittgenstein; Pears; McGuinness, 2001, p. 29).

allows us to further define the remaining constants (disjunction, implication, and equivalence) and build truth tables that, due to soundness and completeness, are equivalent to a calculus. We will refer to this system later.

2.1 Category mistakes

Categories have been traditionally understood as conceptual tools that help us classify objects into partitions according to predication.[1] Following this rather vague description, we say a category system is a theory of categories,[2] an ontology as it were. Thus, category systems are ubiquitous ontological tools that help us classify objects and build taxonomies, and in doing so, they might warn us not to commit category mistakes.

A category mistake occurs when an item belonging to a certain category is assigned an attribute belonging to another category. In *The Concept of Mind*, Ryle coined the term and suggested a *gedankenexperiment* to explain it (Ryle, 1949, p. 12). Suppose an agent visits Oxford for the first time and is shown a number of colleges, libraries, playing fields, museums, scientific departments, and administrative offices. At the end of the visit she asks: "But where is the University? I have seen where the members of the Colleges live, where the Registrar works, where the scientists experiment and the rest. But I have not yet seen the University in which reside and work the members of the University." In doing so she inserts the University, an item belonging to the category of institutions, into the category of buildings, thus conflating ontological categories, hence committing a category mistake.

2.2 Reasoning and category mistakes

Granted, the previous example does not seem to be too fascinating; but the issue becomes more interesting once we try to analyze complex reasonings. Consider that, ordinarily, reasonings are a rational products that require not only valid forms, but also true premises in order to obtain admissible grounds for the acceptance of a conclusion. But it is quite easy to find infinitely many instances of sound reasonings (i.e., valid forms with true contents) that do not seem to be rhetorically acceptable. Consider a toy example.

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1 Although there are antecedents in Aeschylus (*Seven against Thebes*, 439), Herodotus (*Book III*, 71) and Plato (*Sophist*), Aristotle is arguably the first one to explicitly develop a category system in this sense (*Categories*, IV 1 b 26; *Topics* 107; *Metaphysics* 1016b). Medieval logicians, like Peter of Spain, William of Sherwood, William of Occam, and Albert of Saxony, also shared their part through theories of supposition (Boehner, 1952). Modernity also provided category systems, being Kant's probably the most famous (*Critique of Pure Reason* A70/B95). And needless to say, category systems are also present in contemporary philosophy, both in the analytical and the continental traditions (Russell and North Whitehead, 1913; Husserl, 1913; Ingarden, 1964; Johansson, 1989; Chisholm, 1989; Hoffman and Rosenkrantz, 1994; Lowe, 2006).

2 We have deliberately chosen the expression "theory of categories" to distinguish our notion from the mathematical notion of "category theory."

Example 1 *If the "sun" is monosyllabic, the sun exists. In fact, "sun" is monosyllabic. Therefore, the sun exists.*

Example 1 is clearly sound in the previous sense because it is valid—it is an instance of a *Modus Ponens*—and its premises are true; however, it is also clear that it is counter-intuitive and not rhetorically acceptable. Now, it is true that, from a pragmatic point of view, we would be inclined to conclude, rather simply, that the fact that the word "sun" is monosyllabic has nothing to do with the existence of the sun, the star. But pragmatic solutions of this sort, albeit useful, are not systematic nor well defined (Cf. Mares, 2004, p. 26-27).

It is also true that we could attempt to reject instances of Example 1 on the basis of logical criteria defined after some relevant semantics, for example, after situation semantics for relevant logic (Mares, 2004). According to this semantics, Example 1 would look like a relevant *Modus Ponens* in which situations are denoted by subsets: $((A\supset B_{\{1\}})\land A_{\{2\}})\supset B_{\{1,2\}}$. This attempted solution, however, will not suffice. For one, there is nothing in such semantics that prevent us from detaching B from the conjunction of $A\supset B$ and A, since such subsets are not ordered, and thus, this solution offers no answer to the problem. Moreover, if we consider a more complex, but still toy, example (Example 2), it is not clear how a relevant approach would be of more use than the classical propositional procedure.

Example 2 *John belief in φ implies that ψ is possible, and if Paul knows γ then it is a fact that γ. Now, γ is actually necessary unless φ is also a fact. Therefore, ψ is not only a belief, but a fact.*

Unlike Example 1, this second one is way more complex, and consequently, requires a more complex analysis because it seemingly demands a modal approach. Alas, there is a couple of drawbacks with this solution: it would make the analysis computationally more complex due to the fact that it would require a mixture of semantics for belief, knowledge, possibility, and necessity; and yet, it would not be able to rule out Example 1 as an unacceptable reasoning.

Consequently, since these approaches are not able to give a sound account of Examples 1 and 2, and since we think the problem of Example 1 is due to a category mistake, and that a simple analysis of Example 2 can be achieved with the notion of category system, our commitment is to develop a formal tool that comprises these ideas.

3 Sketch of a logical system

3.1 Scalar logic L_c

In this section we sketch a scalar logical system designed to deal with categories, we call it L_c. Its vocabulary includes constants, CONS=$\{\neg, \land, \lor, \supset, \equiv\}$, and variables, VAR=$\{\varphi, \psi, \ldots\}$; its syntax, two rules: *i)* if $\varphi\in$VAR, φ is a wff of L_c; *ii)* if φ and ψ are wffs of L_c, then $\neg\varphi$, $\varphi\land\psi$, $\varphi\lor\psi$, $\varphi\supset\varphi$, and $\varphi\equiv\psi$ are wffs of L_c. Finally, its semantics requires some extra assumptions. The first one is that the set of categories is numerable. With this assumption in mind, we posit that L_c supports the next five models:

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A) An ontological-linguistic model that represents the relation between the category of concepts and the category of facts.
B) An epistemic-doxastic model that represents the relation between the category of knowledge and the category of beliefs.
C) An aletic model that represents the relation between the category of necessity and the category of possibility.
D) A deontological model that represents the relation between the category of obligation and the category of permission.[1]
E) A combined model that relates models A, B, and C.

In this contribution we focus only on model E. To explain its semantics, consider the next definitions.

Definition 1 (Assignment) An assignment in L_c is a function f:VAR™{⊤, ⊥} that maps the variables to a multiset where ⊥ is the anti-designated value, and ⊤ denotes the designated values {ς, π, δ, ο, ε, ν} that represent the categories of model E: concepts, possibilities, beliefs, facts, knowledge, and necessities, respectively.

Definition 2 (Valuation) Given an assignment f, a valuation v_{Lc}, for any wffs φ and ψ, verifies: *i)* $v_{Lc}(\varphi)=f(\varphi)$, *ii)* $v_{Lc}(\neg\varphi)=v_{Lc}(\varphi)$, and *iii)* $v_{Lc}(\varphi*\psi)=*(v_{Lc}(\varphi),v_{Lc}(\psi))$, for $*\in$CONS/¬.

With this valuation, we can proceed to define the constants. For sake of exposition, assume the next replacements in the set ⊤: to {ς}, assign number 4; to {π, δ}, number 3; to {ο, ε}, number 2; and to {ν}, number 1.

Definition 3 (Negation) The negation of a wff φ (¬φ) is defined by an idempotent function, $v_{Lc}(\neg\varphi)=v_{Lc}(\varphi)$, that defines the next matrix:

φ	¬φ
⊥	⊥
1	1
2	2
3	3
4	4

Definition 4 (Conjunction) The conjunction of two wffs φ and ψ (φ∧ψ) is defined by a minimum function:

$$v_{Lc}(\varphi\wedge\psi) = \begin{cases} \min(v_{Lc}(\varphi),v_{Lc}(\psi)), \text{ if } v_{Lc}(\varphi),v_{Lc}(\psi)\in\top \\ \bot, \text{ otherwise} \end{cases}$$

1 This model will be avoided during the current exposition because deontology posits philosophical problems that would require more space than we have.

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that defines the next matrix:

∧	⊥	1	2	3	4
⊥	⊥	⊥	⊥	⊥	⊥
1	⊥	1	1	1	1
2	⊥	1	2	2	2
3	⊥	1	2	3	3
4	⊥	1	2	3	4

Definition 5 (Disjunction) The disjunction of two wffs φ and ψ (φ∨ψ) is defined by a maximum function:

$$v_{Lc}(\varphi \vee \psi) = \begin{cases} \max(v_{Lc}(\varphi), v_{Lc}(\psi)), \text{ if } v_{Lc}(\varphi), v_{Lc}(\psi) \in \top \\ \bot, \text{ otherwise} \end{cases}$$

that defines the next matrix:

∨	⊥	1	2	3	4
⊥	⊥	⊥	⊥	⊥	⊥
1	⊥	1	2	3	4
2	⊥	2	2	3	4
3	⊥	3	3	3	4
4	⊥	4	4	4	4

Definition 6 (Implication) The implication of two wffs φ and ψ (φ⊃ψ) is defined by an order relation:

$$v_{Lc}(\varphi \supset \psi) = \begin{cases} v_{Lc}(\varphi), \text{ if } v_{Lc}(\varphi) \leq v_{Lc}(\psi) \text{ and } v_{Lc}(\varphi), v_{Lc}(\psi) \in \top \\ \bot, \text{ otherwise} \end{cases}$$

that defines the next matrix:

⊃	⊥	1	2	3	4
⊥	⊥	⊥	⊥	⊥	⊥
1	⊥	1	1	1	1

2	⊥	⊥	2	2	2
3	⊥	⊥	⊥	3	3
4	⊥	⊥	⊥	⊥	4

Definition 7 (Equivalence) The equivalence of two wffs φ and ψ (φ≡ψ) is defined by an identity function:

$$v_{Lc}(\varphi \equiv \psi) = \begin{cases} v_{Lc}(\varphi), & \text{if } v_{Lc}(\varphi)=v_{Lc}(\psi) \text{ and } v_{Lc}(\varphi), v_{Lc}(\psi) \in \top \\ \bot, & \text{otherwise} \end{cases}$$

that defines the next matrix:

≡	⊥	1	2	3	4
⊥	⊥	⊥	⊥	⊥	⊥
1	⊥	1	⊥	⊥	⊥
2	⊥	⊥	2	⊥	⊥
3	⊥	⊥	⊥	3	⊥
4	⊥	⊥	⊥	⊥	4

Given this semantics, we can build a *category matrix*, a device analogous to a truth table (Example 3) .

Example 3 Consider the wff ((φ∨ψ)∧¬φ)⊃ψ. Its category matrix is the next one (for sake of brevity, we omit the combinations starting with ⊥).

((φ	∨	ψ)	∧	¬φ)	⊃	ψ
1	1	1	1	1	1	1
1	2	2	1	1	1	2
1	3	3	1	1	1	3
1	4	4	1	1	1	4
2	2	1	2	2	⊥	1
2	2	2	2	2	2	2
2	3	3	2	2	2	3
2	4	4	2	2	2	4
3	3	1	3	3	⊥	1
3	3	2	3	3	⊥	2

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3	3	3	3	3	3	3
3	4	4	3	3	3	4
4	4	1	4	4	$\perp$	1
4	4	2	4	4	$\perp$	2
4	4	3	4	4	$\perp$	3
4	4	4	4	4	4	4

The category matrix above shows that the wff $((\varphi\vee\psi)\wedge\neg\varphi)\supset\psi$ is not valid in L_c because it commits a category mistake (a fact denoted by the rows with an anti-designated value, $\perp$). This observation, as expected, allows us to introduce the fundamental concepts of tautology, contradiction, and contingency for categories. The concept of validity, of course, would be abbreviated as usual. The concept of category mistake, on the other hand, would be the category counter-part of a contradiction.

Definition 8 (Tautology$_{Lc}$) A tautology$_{Lc}$ is a wff φ s.t. for all valuations v_{Lc}, $v_{Lc}(\varphi)\in\top$, i.e., it is a wff with no category mistakes.

Definition 9 (Contradiction$_{Lc}$) A contradiction$_{Lc}$ is a wff φ s.t. for all valuations v_{Lc}, $v_{Lc}(\varphi)=\perp$, i.e., it is a wff that always commits a category mistake.

Definition 10 (Contingency$_{Lc}$) A contingency$_{Lc}$ is a wff that is not a tautology$_{Lc}$ nor a contradiction$_{Lc}$.

With the system sketched so far, we can observe that it provides us with a general ontology for reasoning with 6 categories (Figure 1) (or 8 categories, if we add model 4 (Figure 2)) that defines a category system for general reasoning.

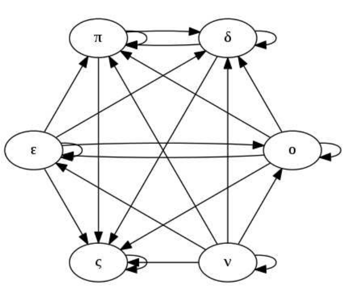


Fig. 1 An ontology with 6 categories

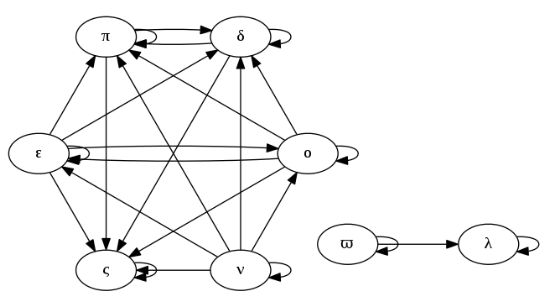


Fig. 2 An ontology with 8 categories

3.2 Product logic $L_{c\times p}$

However, for reasons that will become evident in a moment, we need to introduce the system $L_{c\times p}$ as the product logic obtained from multiplying scalar logics L_c and L_p as defined previously. Consider, for a start, the following basic definitions.

Definition 11 (Tautology$_{Lcxp}$) A tautology$_{Lcxp}$ is a wff φ s.t. for all valuations v_{Lc} and v_{Lp}, $v_{Lc}(\varphi) \in \top$ and $v_{Lp}(\varphi)=1$, where 1 represents the designated value of L_p.

Definition 12 (Contradiction$_{Lcxp}$) A contradiction$_{Lcxp}$ is a wff φ s.t. for all valuations v_{Lc} and v_{Lp}, $v_{Lc}(\varphi)=\bot$ and $v_{Lp}(\varphi)=0$, where 0 represents the anti-designated value of L_p.

Definition 13 (Contingency$_{Lcxp}$) A contingency$_{Lcxp}$ is a wff that is not a tautology$_{Lcxp}$ nor a contradiction$_{Lcxp}$.

In $L_{c\times p}$, the values assigned to a wff look like ordered products of the form α×β, where $\alpha \in \{\top, \bot\}$ and $\beta \in \{1, 0\}$. To illustrate how this works, consider the next examples.

Example 4 Recall Example 1: *If the "sun" is monosyllabic, the sun exists. In fact, "sun" is monosyllabic. Therefore, the sun exists*. Let *A* stand for "*The "sun" is monosyllabic*" and let *B* stand for "*The sun exists*". Now, according to the categories involved, *A* would be talking about properties of concepts (i.e., it would belong to category 4) and *B* would be talking about facts (i.e., it would belong to category 2). Consequently, Example 1 would look like a *Modus Ponens* with the corresponding categories attached as sub-indexes: $((A_4 \supset B_2) \wedge A_4) \supset B_2$. Figure 3 clearly shows that, although such reasoning has true premises and is valid-L_p (since it is an instance of a *Modus Ponens*), it commits a category mistake in L_c (i.e., it is invalid-L_c), which explains why it is not rhetorically acceptable without retorting to pragmatic solutions, relevant semantics or modal techniques.

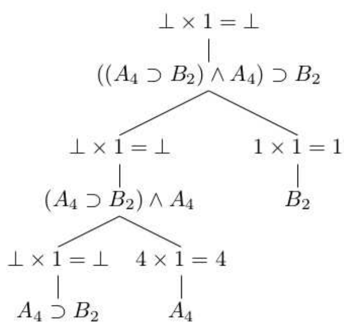


Fig. 3 Example of a valid-L_p but invalid-L_c reasoning

Hence, notice that if we were to consider the cogency of Example 1 (4) solely on L_c grounds, we would obtain an invalid-L_c reasoning; but if we were to consider

Example 1 (4) only by the tenets of L_p, we would obtain a valid-L_p reasoning. This observation allows us to infer the next combinations of (in)validity for $L_{c\times p}$ (Table 1).

valid-L_c	x	valid-L_p	=	valid-$L_{c\times p}$
valid-L_c	x	invalid-L_p	=	invalid-$L_{c\times p}$
invalid-L_c	x	valid-L_p	=	invalid-$L_{c\times p}$
invalid-L_c	x	invalid-L_p	=	invalid-$L_{c\times p}$

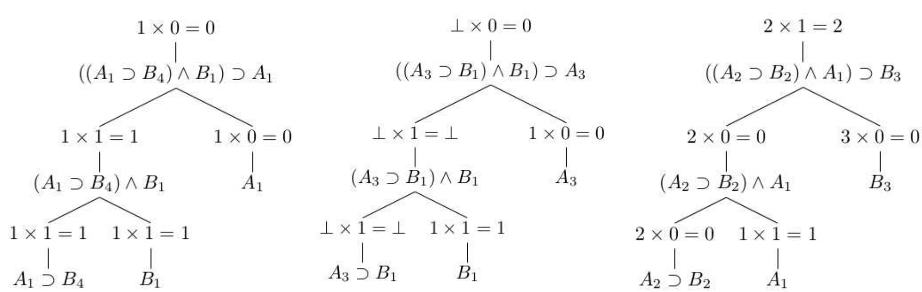
Table 1. Combinations of (in)validity for $L_{c\times p}$

Example 4 shows the invalid-$L_c$xvalid-L_p case. To illustrate the remaining combinations, consider the next examples.

Example 5 Suppose a valid-L_c but invalid-L_p reasoning, say, a Fallacy of Affirmation of the Consequent such that $((A_1 \supset B_4) \wedge B_1) \supset A_1$. This reasoning turns out to be invalid-$L_{c\times p}$ (Figure 4a).

Example 6 Now consider a reasoning both invalid-L_c and invalid-L_p: $((A_3 \supset B_1) \wedge B_1) \supset A_3$. This reasoning is invalid-$L_{c\times p}$ (Figure 4b).

Example 7 Finally, consider a reasoning both valid-L_c and valid-L_p: $((A_2 \supset B_2) \wedge A_1) \supset B_3$. This reasoning turns out to be valid-$L_{c\times p}$ (Figure 4c).



a) valid_{Lc}xinvalid_{Lp} = invalid_{Lcxp} b) invalid_{Lc}xinvalid_{Lp} = invalid_{Lcxp} c) valid_{Lc}xvalid_{Lp}=valid_{Lcxp}

Fig. 4 Combinations of (in)validity for $L_{c\times p}$

3.3 A calculus for $L_{c\times p}$

At this point, it should be evident the computational complexity of $L_{c\times p}$ compromises its usefulness. Therefore, in order to facilitate reasoning with it, we define some rules following a natural deduction style.

Definition 14 (Deduction) Let Γ be a set of wffs of $L_{c\times p}$ and φ a wff of L_{cxp}. $\Gamma\vdash_{Lc\times p}\varphi$ is a deduction of φ from Γ in $L_{c\times p}$ if and only if *i)* φ∈Γ or *ii)* φ is obtained from previous members of Γ by applying a $L_{c\times p}$ rule.

Before we introduce such rules, we must comment on four features about notation. First, where inference is valid for any category, we omit the sub-indexes; second, when we use the expression $(\varphi*\psi)_i$ we mean that the value of such wff is category *i*; third, we use F to stand for a wff of the form (φ∧¬φ); and fourth, the notation [φ] indicates that φ is an assumption.

	Introduction		Elimination	
∧	$\dfrac{\varphi \qquad \psi}{\varphi\wedge\psi}$		$\dfrac{\varphi\wedge\psi}{\varphi}$	
∨	$\dfrac{\varphi_i}{\varphi_i\vee\psi_{j\leq i}}$		$\dfrac{(\varphi\supset\gamma)_i \quad (\psi\supset\gamma)_j \quad (\varphi\vee\psi)_{k\leq i,j}}{\gamma_{j\leq i}}$	$\dfrac{(\varphi\vee\psi)_i \quad \neg\varphi_{j\leq i}}{\psi_{j\leq i}}$
⊃	$\dfrac{[\varphi_i] \; \dots \; \psi_j}{(\varphi\supset\psi)_{i\leq j}}$	$\dfrac{(\varphi\supset\gamma)_{i\leq j} \quad (\gamma\supset\psi)_{j\leq k}}{(\varphi\supset\gamma)_{i\leq k}}$	$\dfrac{(\varphi\supset\psi)_i \quad \varphi_{j\leq i}}{\psi_{j\leq i}}$	$\dfrac{(\varphi\supset\psi)_i \quad \neg\psi_{j\leq i}}{\neg\varphi_{j\leq i}}$

Further, we have a couple of inference rules governing F:

$\dfrac{F}{\varphi}$	$\dfrac{[\neg\varphi] \; \dots \; F}{\varphi}$

3.4 Logical properties of $L_{c\times p}$

Given these elements, we now sketch proofs of soundness, consistency, and completeness with respect to the inference rules.

Proposition 1 (Validity preservation) $L_{c\times p}$ inference rules are tautologies-$L_{c\times p}$.

If we perform the category matrix of each inference rule, we obtain valid-L_{cxp} expressions, i.e., tautologies in L_{cxp}. The goal of this first proposition is to allow the preservation of L_{cxp}-validity from premises to conclusions with respect to the combinations of categories.

Proposition 2 (Soundness) If $\Gamma\vdash_{Lc\times p}\varphi$, then $\Gamma\vDash_{Lc\times p}\varphi$.

To sketch a proof of this statement imagine an induction on the size of the deduction of φ. For the base case, if Γ has just one element, then φ∈Γ, in which case clearly $\Gamma\vDash_{L_{c\times p}}\varphi$. For the inductive case, consider an arbitrary wff φ_j obtained by inference rules in less than *n* steps from previous formulas in Γ. For the case of elimination rules, such wffs would have to be of the form $\psi_{j\leq i}$ and $(\psi*\varphi)_i$, for $*\in$CONS/¬. Then, by the induction hypothesis, both $\Gamma\vDash_{L_{c\times p}}\psi_{j\leq i}$ and $\Gamma\vDash_{L_{c\times p}}(\psi*\varphi)_i$, and since each inference rule is valid-$L_{c\times p}$, if we apply a rule concerning *, we can deduce $\varphi_{j\leq i}$ in *n* steps and thus $\Gamma\vDash_{L_{c\times p}}\varphi_{j\leq i}$. For the case of introduction rules, an arbitrary wff $(\psi*\varphi)_i$, for $*\in$CONS/¬, has to be obtained in less than *n* steps from previous wffs of the form $\psi_{i\leq j}$ and $\varphi_{j\leq k}$. Then, by the induction hypothesis, $\Gamma\vDash_{L_{c\times p}}\psi_{i\leq j}$ and $\Gamma\vDash_{L_{c\times p}}\varphi_{j\leq k}$, and since each inference rule is valid-$L_{c\times p}$, if we apply a rule concerning *, we can deduce $(\psi*\varphi)_{i\leq j}$ in *n* steps and thus $\Gamma\vDash_{L_{c\times p}}(\psi*\varphi)_{i\leq j}$. Now, for the last couple of rules, φ has to be obtained in less than *n* steps from F or from $F\cup\{\neg\varphi\}$. In the first case, $F\in\Gamma$, so that $\Gamma\vDash_{L_{c\times p}}F$, but since $v_{Lcxp}(F)=\bot\times 0$ for all valuations, there is no valuation $v_{Lcxp}(\psi)=\top\times 1$ for any $\psi\in\Gamma$. Now let $\Gamma'=\Gamma\cup F$ but assume $\Gamma'\nvDash_{Lcxp}\varphi$, then $v_{Lcxp}(\psi)=\top\times 1$ for any $\psi\in\Gamma'$ and $v_{Lcxp}(\varphi)=\bot\times 0$ for some valuation; but if Γ' includes Γ∪F we obtain a contradiction. For the remaining case, consider that Γ includes F and ¬φ. Let $\Gamma'=\Gamma\cup F$ and suppose $\Gamma'\nvDash_{Lcxp}\varphi$, then there exists a valuation $v_{Lcxp}(\psi)=\top\times 1$ for any $\psi\in\Gamma'$ but $v_{Lcxp}(\varphi)=\bot\times 0$. Now let $\Gamma''=\Gamma'\cup\{\neg\varphi\}$, so Γ'' contains the deduction from Γ' so that $v_{Lcxp}(\psi)=\top\times 1$ for any $\psi\in\Gamma''$, but that is impossible since $\Gamma''\vDash_{L_{c\times p}}F$, and thus, $\Gamma\vDash_{L_{c\times p}}\varphi$.

Proposition 3 (Consistency) It is not the case that $\Gamma\vDash_{L_{c\times p}}\varphi$ and $\Gamma\vDash_{L_{c\times p}}\neg\varphi$.

To sketch a proof suppose, for *reductio*, that both $\Gamma\vdash_{L_{c\times p}}\varphi$ and $\Gamma\vdash_{L_{c\times p}}\neg\varphi$ hold. Then, by Proposition 2, both $\Gamma\vDash_{L_{c\times p}}\varphi$ and $\Gamma\vDash_{L_{c\times p}}\neg\varphi$. Hence, for every $v_{L_{c\times p}}$, $v_{L_{c\times p}}(\varphi)=\top\times 1$ and $v_{L_{c\times p}}(\neg\varphi)=\top\times 1$, which is absurd.

Proposition 4 (Completeness) If $\Gamma\vDash_{L_{c\times p}}\varphi$, then $\Gamma\vdash_{L_{c\times p}}\varphi$.

Now, in order to sketch completeness, suppose the next three auxiliary remarks hold. First, that an extension L^+ is a logical system obtained from another system L by adding a new rule in such a way that all the rules of L remain the same (it should be clear that an extension L^+ is consistent if there is a wff that is not a theorem of L^+). Second, that an extension is complete if for every wff φ, either φ or ¬φ is a theorem of the extension. And third, that if L^+ is a consistent complete extension, there is a valuation v_{L+} in which every inference rule of L^+ is valid. So, suppose that $\Gamma\vDash_{L_{c\times p}}\varphi$ (i.e., $v_{L_{c\times p}}(\varphi)=\top\times 1$ for all $v_{L_{c\times p}}$ valuations) but $\Gamma\nvdash_{L_{c\times p}}\varphi$. If this is the case, the extension L^+ that results from $L_{c\times p}$ by adding ¬φ must also be consistent and complete, by the first and the second remarks. Hence, there must be a valuation v_{L+} so that for all $\varphi\in L^+$, $v_{L+}(\neg\varphi)=\top\times 1$; in particular, $v_{L_{c\times p}}(\neg\varphi)=\top\times 1$, but by assumption, $v_{L_{c\times p}}(\varphi)=\top\times 1$, which is a contradiction.

4 Conclusions

We have sketched a propositional logical system designed to represent and evaluate reasoning with philosophical categories. We believe this should be of relative interest because, during the development of the system, we attempted to formalize the informal notion of category mistake; and because, with the system thus developed, we provided an alternative to regulate reasoning involving categories.

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