AUFKLÄRUNG, João Pessoa, v.10, n.3, Set.-Dez., 2023, p.89-100
DOI: https://doi.org/10.18012/arf.v10i3.66174
Recebido: 30/03/2023 | Aceito: 30/09/2023

The reduction rules are easy even within a Popperian view

[As regras de redução são fáceis mesmo dentro de uma visão popperiana]

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Abstract: Rudolf Carnap proposed three reduction rules to improve scientific language. Those rules indicate when it is correct to add a property in situations in which other properties are already had. Assuming the theory of mental models, it has been shown that the rules are not hard to use from the cognitive point of view. The key to argue that is to accept that the human mind works as dual-process theories claim. According to these theories, people can use two different systems. One of them implies effort but the other one does not. Thus, the idea is that Carnap's reduction rules can be applied resorting just to the system not implying cognitive effort. This paper goes one step further and poses that even from a Popperian perspective the reduction rules keep not being difficult. One might think that falsifiability requires cognitive effort because it needs to address situations in which the sentences are false. However, this paper tries to explain that, following the theory of mental models, that is not hard if the sentences are conditionals, and the sentences in the reduction rules have conditional structures.
Keywords: Carnap; falsifiability; mental models; Popper; reduction

Resumo: Rudolf Carnap propôs três regras de redução para melhorar a linguagem científica. Essas regras indicam quando é correto adicionar uma propriedade em situações em que outras propriedades já são tidas. Assumindo a teoria dos modelos mentais, foi demonstrado que as regras não são difíceis de usar do ponto de vista cognitivo. A chave para argumentar isso é aceitar que a mente humana funciona como afirmam as teorias de processo dual. De acordo com essas teorias, as pessoas podem usar dois sistemas diferentes. Um deles implica esforço, mas o outro não. Assim, a ideia é que as regras de redução de Carnap podem ser aplicadas recorrendo apenas ao sistema não implicando esforço cognitivo. Este artigo vai um passo além e propõe que, mesmo sob uma perspectiva popperiana, as regras de redução continuam não sendo difíceis. Pode-se pensar que a falseabilidade requer esforço cognitivo porque precisa abordar situações em que as sentenças são falsas. No entanto, este artigo tenta explicar que, seguindo a teoria dos modelos mentais, isso não é difícil se as sentenças são condicionais, e as sentenças nas regras de redução têm estruturas condicionais.
Palavras-chave: Carnap; falseabilidade; modelos mentais; Popper; redução

Introduction

Rudolf Carnap (1936) proposed three rules to build scientific theories. They are three reduction rules that indicate the cases in which new properties can be added to scientific definitions. In particular, they show the relations between the old and the new properties allowing the new properties to be included in definitions. This is

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a part of Carnap's project to construct a suitable scientific language (see also, e.g., CARNAP, 1937). However, the point of this paper is that it has been shown that, if a contemporary cognitive framework, that is, the theory of mental models (e.g., KHEMLANI; BYRNE; JOHNSON-LAIRD, 2018), is assumed, the three reduction rules seem not to require cognitive effort for researchers (LÓPEZ-ASTORGA, 2021).

The argument is based on an idea the theory of mental models shares with other theories: the situations when people reason can be two. They might make cognitive effort and they might not (see also, e.g., JOHNSON-LAIRD; KEMLANI; GOODWIN, 2015). The inferences that can be made in each of those two cases are different. Nevertheless, following the theory of mental models, the reduction rules can lead to the necessary conclusions without a lot of effort (LÓPEZ-ASTORGA, 2021). The key is that the logical forms of the reduction rules are conditional structures. According to the theory of mental models, conditionals are generally linked to three possibilities, which, in principle, appear to match the situations in which the material conditional is true in classical propositional calculus (although, as pointed out below, they do not actually). Nonetheless, when individuals make little effort, they are only able to note one of those situations: the situation in which both what the antecedent describes and what the consequent indicates are true (see also, e.g., QUELHAS; RASGA; JOHNSON-LAIRD, 2017). This last possibility is the only one required to apply the reduction rules. So, the rules are not hard for the human mind.

However, that argumentation can be challenged from a Popperian position. Following Popper (e.g., POPPER, 1963), it is necessary to consider situations in which the hypotheses are false. For this reason, one might think that, from the Popperian perspective, to provisionally accept sentences derived from the reduction rules, the researcher has to take into account circumstances in which the clauses of those sentences do not happen. This, according to the theory of mental models, can mean the need for cognitive effort (see also, e.g., JOHNSON-LAIRD; RAGNI, 2019). Thus, working with reduction rules would be hard, at least if Popper's philosophy (see also, e.g., POPPER, 2002) is assumed.

The point this paper tries to make is that this is not necessarily the case. It is possible to accept the Popperian framework and the theory of mental models, and, at the same time, to claim that the reduction rules are cognitively easy. In this way, the sections of the present paper will be the following.

The first section will show what, in Carnap's (1936) view, the reduction rules and its restrictions are. The second one will describe the manner the theory of mental models understands conditionals and the cognitive effort that, according to the theory, should be made regarding that kind of sentence. The next section will explain why, from the theory of mental models, it can be stated that the reduction rules are not difficult for human beings. An example, focused on the first reduction rule, will be developed in this sense. The fourth section will reveal why Popper's proposal seems to be a challenge for the account from the theory of mental models. The final section will try to argue that challenge can be overcome. The idea will be that, even if the Popperian approach is assumed, the theory of mental models keeps making it evident that the reduction rules are not hard to use.

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Three reduction rules

The rules Carnap (1936) provides are these ones:

(R) $P \to (Q \to R)$

(R_1) $P \to (Q \to R)$
(R_2) $S \to (T \to \neg R)$

(R_b) $P \to (Q \leftrightarrow R)$

Sentences (R), (R_1) and (R_2), and (R_b) are under the scope of universal quantifiers. Therefore, P, Q, R, S, and T are predicates. Carnap (1936) expresses them with other symbols. Here, '$\to$' stands for conditional relation, '$\neg$' represents negation, and '$\leftrightarrow$' indicates biconditional relation.

These rules refer to situations in which what the researcher wants to do is to clarify the meaning of a predicate such as R. According to Carnap (1936, pp. 442-443), (R) is a "reduction sentence" for R, (R_1) and (R_2) are a "reduction pair" for R, and (R_b) is a "bilateral reduction sentence" for R. Nevertheless, another important point is that (R_b) captures the circumstance in which (1) and (2) occur.

$S \leftrightarrow P$

$T \leftrightarrow \neg Q$

Thereby, Carnap (1936) explains that another way to present (R_2) can be (3).

$P \to (\neg Q \to \neg R)$

And (R_1) and (3) allow coming to (R_b) (See also, e.g., LÓPEZ-ASTORGA, 2021).

Nonetheless, Carnap's (1936) account is not that simple. (R), (R_1), (R_2), and (R_b) cannot always be applied. There are restrictions, which are these ones (they are also expressed here with symbols different from those that CARNAP, 1936, uses):

For (R),

$\neg(P \wedge Q)$ cannot be the case

Where '$\wedge$' stands for conjunction.

For (R_1) and (R_2),

$\neg[(P \wedge Q) \vee (S \wedge T)]$ cannot be the case

Where '$\vee$' refers to disjunction.

And for (R_b),

$\neg$P cannot be the case

Restrictions (4), (5), and (6) are important in the argumentation that the reduction rules are cognitively easy in accordance with the theory of mental models. Before addressing this issue, the next section is devoted to the account this theory gives about the conditional.

Cognitive effort and the conditional in the theory of mental models

The theory of mental models proposes that people process 'sentential connectives' as 'conjunctions of possibilities' (see also, e.g., KHEMLANI; HINTERECKER; JOHNSON-LAIRD, 2017). Given that the relevant connective for this paper is the conditional, this section will focus on sentences such as (7).

$A \rightarrow B$

According to the theory of mental models, when individuals find a sentence such as (7) in natural language, they tend to consider three possibilities related by means of conjunctions (see also, e.g., LÓPEZ-ASTORGA; RAGNI; JOHNSON-LAIRD, 2022). (8) expresses this.

$\lozenge (A \wedge B) \wedge \lozenge (\neg A \wedge B) \wedge \lozenge (\neg A \wedge \neg B)$

Symbol '$\lozenge$' represents possibility. But it is not here the usual symbol in normal modal logic. Several components differentiate the theory of mental models from normal modal logics. Just an example is that in the theory of mental models (9) can be derived from (7) (for an explanation akin to the one below, see, in addition, e.g., ESPINO; BYRNE; JOHNSON-LAIRD, 2020).

$\lozenge A$

The reason is that (8) corresponds to (7), and the first possibility in (8) is (10).

$\lozenge (A \wedge B)$

And A is possible in (10).

However, this derivation is not admitted in normal modal logics. The conditional is material in normal modal logic. Hence, as it is well known, under the material interpretation of the conditional, a sentence such as (7) can be the case even if its antecedent, in this case, A, is impossible (see, e.g., RESTALL, 2006). Accordingly, if (7) is true, that does not imply that (9) is true too.

Nevertheless, a relevant component of the theory of mental models for this paper is that it adopts the general framework of the dual-process theories (e.g., REYNA, 2004). These theories state that, when people reason, they might make effort or they might not (see also, e.g., STANOVICH, 2012). Within the theory of mental models and for the particular case of (7), this can lead to two different situations. If there is no effort, people only detect the first model in (8), that is, (10). The other two models, that is, conjunction (11), are only identified when the effort made is significant (see also, e.g., BYRNE; JOHNSON-LAIRD, 2020).

$$\Diamond(\neg A \wedge B) \wedge \Diamond(\neg A \wedge \neg B)$$

On the other hand, the possibilities in (11) are deemed in the theory as presuppositions (e.g., ESPINO ET AL., 2020). Thus, they are possibilities attributable to (7) both when (7) is true and when (7) is false (see also, e.g., KHEMLANI; ORENES; JOHNSON-LAIRD, 2014). This is another point in which the theory of mental models moves away from logic. It is possible that people understand that the negation of (7) is, as in classical logic and following the material interpretation of the conditional, (12).

$$A \wedge \neg B$$

Nonetheless, several experimental results (e.g., KHENLANI ET AL., 2014) suggest that individuals can also interpret the negation of (7) to be equivalent to (13).

$$A \rightarrow \neg B$$

This aspect of the theory is interesting because it leads to the idea that (11) represents two presuppositions requiring cognitive effort and being valid both for the case in which the conditional happens and for the case in which it does not happen. If the negation of (7) is understood as (13), then the possibility that can be recovered effortlessly is (14).

$$\Diamond(A \wedge \neg B)$$

But if the necessary effort were made, the conjunction of possibilities would be (14) together with (11), that is, (15) (see also, e.g., LÓPEZ-ASTORGA ET AL., 2022).

$$\Diamond(A \wedge \neg B) \wedge \Diamond(\neg A \wedge B) \wedge \Diamond(\neg A \wedge \neg B)$$

Which reveals that if the negation of (7) is interpreted as (13), the only model that is not permitted is (10) (see also, e.g., BYRNE; JOHNSON-LAIRD, 2020).

All of this enables to show that (R), (R_1), (R_2), and (Rb) can be used without making effort. That has already been addressed in the literature (LÓPEZ-ASTORGA, 2021). The next section deals with the case of (R) as an example.

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Reduction sentences and cognitive effort

It is easy to note why the application of (R) does not imply cognitive effort. This section will explain the reasons following a previous account in the literature (LÓPEZ-ASTORGA, 2021).

If no effort is made, the possibility that can be derived from (R) is (16).

$$\Diamond[P \wedge (Q \rightarrow R)]$$

Possibility (16) includes a conditional, which is (17).

$$Q \rightarrow R$$

So, (17) should also be processed. If the situation kept without effort, that action would lead to possibility (18).

$$\Diamond(Q \wedge R)$$

In the framework of the theory of mental models, this means that, when individuals do not make effort, they understand (R) as (19).

$$\Diamond[P \wedge \Diamond(Q \wedge R)]$$

Which, given that the theory of mental models is not a modal logic, is equivalent to understand (R) as (20).

$$\Diamond(P \wedge Q \wedge R)$$

Any other possibility different from (20) requires effort. However, restriction (4) removes those possibilities. Cognitive effort could lead to add two possibilities to (16), which are those in (21).

$$\Diamond[\neg P \wedge (Q \rightarrow R)] \wedge \Diamond[\neg P \wedge \neg(Q \rightarrow R)]$$

Nevertheless, the possibilities in (21) should not be considered. In them, P is false. Therefore, they are eliminated by virtue of (4).

On the other hand, cognitive effort would also add the possibilities in (22) to (18).

$$\Diamond(\neg Q \wedge R) \wedge \Diamond(\neg Q \wedge \neg R)$$

But again, the possibilities in (22) cannot be taken into account. In them, Q is not true. For that reason, (4) implies to ignore them.

Accordingly, from the theory of mental models, restriction (4) indicates that the only possibility that should be considered to use (R) is (20), that is, a possibility not needing cognitive effort to be recovered. An explanation akin to that above can be found in the literature (LÓPEZ-ASTORGA, 2021). Similar accounts have been offered for (R_2) and (R_b) too (note that (R_1) is identical to (R); for those accounts, see LÓPEZ-ASTORGA, 2021, as well). Because the accounts for (R_2) and (R_b) are not very different from that described in this section for (R), they will not be reproduced here. From the explanation for (R), it is easy to think about those accounts. What is interesting now is the difficulties the Popperian philosophy could raise for the proposal based on the theory of mental models.

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The Popperian approach and the negation of the conditionals

Popper's framework is a description of how science should work (e.g., POPPER, 2002). Thus, Popper provided a demarcation criterion to differentiate what is scientific and what is not. That criterion was the falsification criterion. To be scientific, a theory needed to be falsifiable. This put theories such as those of Adler, Freud, and Marx out of science (see also POPPER, 1963). Nonetheless, the point that is relevant for this paper is that Popper's criterion implies to deal with the hypothetical cases in which a scientific sentence would be false. The intention would be to check whether or not those cases are possible. Hence, the criterion leads to address situations that would be impossible if the sentence were true.

This is important because it seems that, more or less consciously, the Popperian project was adopted in several branches of science. It has been said even that the adoption is particularly clear in cognitive science (e.g., STENNING; VAN LAMBALGEN, 2004). So, it appears to be appropriate to ask about the consequences that the Popperian theses can have for reduction rules such as those of Carnap. In this way, one might think that, given that Popper's requirements with regard to falsification involve to imagine falsity situations, those requirements can entail cognitive effort to use (R), (R_1), (R_2), and (R_b). Something providing further support to this idea can be the fact that people can negate a conditional in a manner different from that of classical logic and the material interpretation. As said, for the theory of mental models, a conditional such as (7) can be negated understanding (13). That means that (14) is not the only possibility that can correspond to (7) when negated. The possibilities can be those in (15). Thereby, the problem is that the last possibilities in (15), that is, the presuppositions in (11), need effort to be discovered. Therefore, it is possible to think that, if the Popperian position is assumed, (R), (R_1), (R_2), and (R_b) necessitate cognitive effort. But restrictions (4), (5), and (6) reveal that this is not the case.

Reduction, effort, and the Popperian criterion

If some researchers try to falsify (R), they should think about a hypothetical situation in which its main conditional is false. But if people negated conditionals as classical logic indicates, that would be more difficult than it may appear. The possibility to consider would be (23).

$$\Diamond[P \wedge \neg(Q \rightarrow R)]$$

However, to come to (23) is hard. Not only it is necessary to note that (16) is a possibility. It is also required to realize that (R) allows the possibilities in (21). Only in this way individuals could note that the missing possibility, that is, the only possibility (R) does not admit, is (23) (see also, e.g., BARRES; JOHNSON-LAIRD, 2003, where it is shown, by means of experimental results, that, to come to false possibilities, people first think about the true possibilities corresponding to them).

That task cannot be made without effort. Nevertheless, if the negation of (R) is understood as (24),

$$P \rightarrow \neg(Q \rightarrow R)$$

The easy possibility related to (24) is (23).

Likewise, to process the negation of the second conjunct in (23) would also lead to a possibility effortlessly: possibility (23) would be equivalent to (25).

$$\Diamond[P \wedge (Q \rightarrow \neg R)]$$

And, again, with no effort, (25) could be transformed into (26).

$$\Diamond(P \wedge Q \wedge \neg R)$$

Only the presuppositions would require cognitive effort. Those of (24) would be the possibilities in (21). Nonetheless, (21) represents cases of ¬P, which are forbidden by (4). On the other hand, the presuppositions of the conditional in (25) are those in (22). But those presuppositions include ¬Q, which is not allowed by (4) either. Therefore, to falsify (R), only the possibility in (26) needs to be considered. That is a possibility not requiring effort.

The account for (R_1) would be identical to that of (R). As far as (R_2) is concerned, it would be akin too. If (R_2) were false, the possibility to take into account would be (27).

$$\Diamond[S \wedge \neg(T \rightarrow \neg R)]$$

However, as in the case of (23), to come to (27), it is necessary to note first all of the possibilities in which (R_2) is true, that is, possibility (28)

$\Diamond[S \wedge (T \rightarrow \neg R)]$

Plus the presuppositions in (29).

$\Diamond[\neg S \wedge (T \rightarrow \neg R)] \wedge \Diamond[\neg S \wedge \neg(T \rightarrow \neg R)]$

Thus, the missing possibility would be (27), which is that making (R_2) false. As in the previous case, this process would be hard. Nevertheless, if the negation of (R_2) were understood as (30),

$S \rightarrow \neg(T \rightarrow \neg R)$

Possibility (27) would be the possibility that could easily be got. In addition, interpreting the negation of the conditional in this way, (27) would be equivalent to (31).

$\Diamond[S \wedge (T \rightarrow R)]$

Hence, (32) could be identified without effort.

$\Diamond(S \wedge T \wedge R)$

Which would suffice to try to falsify (R_2).

Again, the presuppositions would be the problem. Those of the main conditional in (30) would be those indicated in (33).

$\Diamond[\neg S \wedge \neg(T \rightarrow \neg R)] \wedge \Diamond[\neg S \wedge (T \rightarrow \neg R)]$

Nonetheless, in (33), S cannot be true, and that is forbidden by (5). Regarding the presuppositions of the conditional in (31), they would be:

$\Diamond(\neg T \wedge R) \wedge \Diamond(\neg T \wedge \neg R)$

In them, ¬T appears, which is inconsistent with (5) as well.

Lastly, with regard to (R_b), it comes from (R_1) and (3). Its restriction is, in principle, (6), but, given that it supposes (R_1), (5) also has an influence. It is not necessary to address (R_1) because it is equivalent to (R). Accordingly, the same explanation offered for (R) can be applied to (R_1). Given restriction (4), which is included in restriction (5), the only scenario that can be considered is (26), which does not need further effort. With respect to (3), if the human mind negated conditionals as classical logic provides, it would be necessary, to falsify it, to recover possibility (35).

$\Diamond[P \wedge \neg(\neg Q \rightarrow \neg R)]$

However, as in the cases above, this would require to previously note both possibility (36)

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$\Diamond[P \wedge (\neg Q \rightarrow \neg R)]$

And the presuppositions (37) stands for.

$\Diamond[\neg P \wedge (\neg Q \rightarrow \neg R)] \wedge \Diamond[\neg P \wedge \neg(\neg Q \rightarrow \neg R)]$

In this way, the only missing possibility would be (35).

Once again, the process would be difficult. Nevertheless, the theory of mental models allows coming to (35) directly, since the negation of (3) can be understood as (38).

$P \rightarrow \neg(\neg Q \rightarrow \neg R)$

Furthermore, for the same reason, (35) can be interpreted as (39).

$\Diamond[P \wedge (\neg Q \rightarrow R)]$

Thus, cognitive effort is not needed to come to (40) from (39).

$\Diamond(P \wedge \neg Q \wedge R)$

The effort would only be required for the presuppositions. Those corresponding to (38) are in (41).

$\Diamond[\neg P \wedge \neg(\neg Q \rightarrow \neg R)] \wedge \Diamond[\neg P \wedge (\neg Q \rightarrow \neg R)]$

Nonetheless, in the two presuppositions in (41), P is not the case, and (6) does not allow that.

What seems to be a problem here is the presuppositions of the conditional in (39). (42) represents them.

$\Diamond(Q \wedge R) \wedge \Diamond(Q \wedge \neg R)$

However, the problem is not real. The first possibility in (42) would lead to (20). But, as explained, (20) is easy to detect given (R_1), or (R), which is included in (R_b) (see also, LÓPEZ-ASTORGA, 2021). As far as the second possibility in (42) is concerned, it would transform (39) into (26). Nonetheless, (26) is also easy to obtain by falsifying (R_1), or (R).

So, (R_b) keeps without needing cognitive effort, even if the perspective adopted is the Popperian one.

Conclusions

As essential components to build the scientific language, Carnap (1936) provided three reduction rules: (R), (R_1) and (R_2), and (R_b). The literature showed that, if the theory of mental models is assumed as the framework describing the manner people process information and make inferences, those rules are not hard for the human mind. The key to argue this idea is the distinction the theory proposes between two mental systems. The theory of mental models is presented as a dual-process theory. Therefore, from its view, different effort levels lead to different conclusions.

In the case of the conditional, which is the basic connective of the reduction rules, this means that, when the cognitive effort is not enough, people are only able to think about one scenario in which the conditional can be true: when its two clauses happen. The other two scenarios (those in which the antecedent is negated, the consequent being true in one of them and false in the other one) need more effort to be considered. Nevertheless, restrictions such as (4), (5), and (6) reveal that the cases in which the conditional is true requiring cognitive effort are not necessary to apply the reduction rules. Hence, it can be claimed that the rules can be used effortlessly.

One might raise an objection against this. From a Popperian epistemological position, perhaps effort is required. If researchers try to falsify sentences, they have to take cases in which those sentences are false into account. This appears to demand additional cognitive effort.

However, the main point of the present paper has been to show that is not necessarily correct. According to the theory of mental models, the human mind can negate the conditional in way different from that of classical logic. Individuals can consider the negation of a conditional to be equivalent to another similar conditional in which only one element is distinct: the consequent is negated.

If this is the case, a scenario in which the first clause happens and the second one is false is easy to identify again. What continues to be difficult is the recovery of the possibilities in which the antecedents of the conditionals are false. But the restrictions Carnap (1936) indicates for the reduction rules reveal that those possibilities are not necessary even when the intention is to review the circumstances in which the sentences are false. From this point of view, if the theory of mental models is assumed, even a Popperian philosopher could not affirm that (R), (R_1), (R_2), or (R_b) are hard to apply.

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