OPTIMAL MONETARY RULES IN A CONTEXT OF FISCAL DISEQUILIBRIUM: EVIDENCE FROM BRAZIL (1996:I - 2007:I)

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Abstract: This paper aims to derive an optimal monetary policy rule in a context of fiscal disequilibrium. We analyze the transmission channels of the fiscal and monetary policies through estimation of a Philips curve and the fiscal IS curve. The results indicate that the fiscal deficit is statistically significant and affects the inflation rate indirectly via output gap. Furthermore, the results also suggest a perverse effect of the interest rate on the exchange rate and on inflation. In this context, we found empirical evidence that the Brazilian economy shows a non-Ricardian regime for the period 1996: I to 2007: I.

Key words: Fiscal dominance regime, optimal monetary rule, fiscal desequilibrium.

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1 AN OPTIMAL MONETARY POLICY RULE MODEL

Several emerging countries have adopted the inflation targeting regime in a context of fiscal disequilibrium. Brazil, in spite of the successive primary surplus achieved in the past few years, still presents a high nominal deficit and a worrisome public debt.

Brazilian economists have been concerned with the issue of fiscal dominance and have proposed alternative ways to construct monetary rules, which take into consideration the fiscal constraints in the Brazilian case. Most of them impose a fiscal IS[1] curve which implies an optimal rule of the interest rate reaction also dependent upon the fiscal variable.

The main purpose of this paper is to derive, for Brazil, an optimal monetary policy rule in face of fiscal disequilibrium in the period 1996:I to 2007:I. We intend to analyze the impacts of monetary and fiscal policies on the inflation rate and on the interest rate as follow:

i) Through the transmission channels of the fiscal and monetary policies, by estimating the Phillips curve and a fiscal IS. Here we assess whether or not the fiscal variable is significant and how it affects the inflation rate.

ii) Evaluating the interest rate response to changes in the inflation rate, output gap, fiscal deficit and exchange rate based on an optimal monetary policy rule a la Taylor.

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[1] Typically, in the literature, the IS and Phillips curves are used to obtain an optimal monetary rule. In this context, the denomination "fiscal IS" is justified since a fiscal variable (fiscal deficit) appears in the defining equation. See Freitas and Muinhos (2002) and Verdini (2003).

Our discussion proceeds as follows. In Section II, we derive an optimal monetary policy rule model by assuming a fiscal IS curve. In Section III, we present empirical findings related to the joint fit of the fiscal IS and Phillips curves. Section IV presents the empirical optimal monetary policy rule. Finally, in Section V, we present a summary and the final remarks on our findings.

2 AN OPTIMAL MONETARY POLICY RULE MODEL

Consider the traditional IS curve

$$y_{t+1} = a_1 y_1 + a_2 y_{t-1} + a_3 (R_t - E_t \pi_{t+1}) + a_4 fd_t + a_5 e_t + u_{t+1} \quad (3.1)$$

where y_t denotes the output gap[2], R_t is the nominal interest rate[3], π_t is the inflation rate, fd_t is the fiscal deficit (change of the public debt[4]), e_t is the real exchange rate, and u_t is a demand shock, assumed to be normally distributed[5].

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[2] We define the output gap as the difference between the real current output and the potential output.

[3] We are assuming Fisher's equation, $R_t = r_t + E_t \pi_{t+1}$, where r is the real interest rate.

[4] Change of the ratio public debt, ΔB, is equivalent to nominal public debt (budget deficit).

[5] This model follows the approach of Charles et al. (2003), which is similar to that of Walsh (2003:508-511).

The supply curve (AS curve) is represented by the following Phillips curve

$$\pi_{t+1} = a_6\pi_t + a_7 y_t + a_8\Delta q_t + \eta_{t+1} \tag{1.2}$$

and the equation for the nominal exchange rate q_t is the random walk

$$q_{t+1} = q_t + \vartheta_{t+1}. \tag{1.1}$$

The pass-through effect is $\Delta q_t = q_t - q_{t-1}$. We assume that the shocks η_t, u_t and ϑ_t have zero mean and are uncorrelated.

The expected signs for the parameters are $a_1 > 0$, $a_2 > 0$, $a_3 < 0$, $a_4 > 0$, $a_5 > 0$, $a_6 > 0$, $a_7 > 0$ and $a_8 > 0$.

By taking expectations in Equation (3.2),

$$E_t\pi_{t+1} = a_6\pi_t + a_7 y_t \tag{3.4}$$

and plugging this expression into (3.1), we obtain

$$y_{t+1} = \alpha_1 y_1 + a_2 y_{t-1} + a_3 (R_t - a_6 \pi_t) + a_4 fd_t + a_5 e_t + k_{t+1} \tag{3.5}$$

where $\alpha_1 = a_1 - a_3 a_7$

Policy actions, via control of R_t, affect output and inflation with a one-period lag. At time t, the choice of R_t affects y_{t+1} and π_{t+1}, but y_t, y_{t-1}, π_t, fd_t, e_t and q_t are predetermined.

The state variable at instant t is

$$z_t = a_6\pi_t + a_7 y_t + a_8\Delta q_t \tag{3.6}$$

We define

$$\Theta_t = \alpha_1 y_t + a_2 y_{t-1} + a_3 (R_t - a_6\pi_t) + a_4 fd_t + a_5 e_t \tag{3.7}$$

Equations (3.2) and (3.5) can be rewritten as

$$\pi_{t+1} = z_t + \eta_{t+1} \tag{3.8}$$

and

$$y_{t+1} = \theta_t + u_{t+1} \tag{3.9}$$

We assume that the central bank's loss function is given by

$$L = \frac{1}{2} E_t \sum_{t=1}^{\infty} \beta^{i} [\lambda\, y_{t+i}^2 + \pi_{t+i}^2] \tag{3.10}$$

By taking the expression (3.6) one-step forward and substituting (3.8) and (3.9) into (3.6), we have

$$z_{t+1} = a_6 z_t + a_7 \theta_t + a_8\Delta q_{t+1} + a_6\eta_{t+1} + a_7 u_{t+1} \tag{3.11}$$

The policymaker objective is to minimize the loss function (3.10) subject to (3.11). Hence, we can define the value function as

$$V(z_t) = \min_{\theta_t} E_t\left[\frac{1}{2}(\lambda\, y_{t+1}^2 + \pi_{t+1}^2) + \beta V(z_{t+1})\right] \tag{3.12}$$

By substituting (3.8), (3.9) and (3.11) into (3.12), we get

$$V(z_t) = \min_{\theta_t}\left[\frac{1}{2}\lambda \quad E_t(\theta_t + u_{t+1})^2 + \frac{1}{2}E_t(z_t + \eta_{t+1})^2 + \beta \quad E_t V(a_6 z_t + a_7 \quad \theta_t + a_8 \Delta q_{t+1} + a_7 \quad u_{t+1} + a_6 \eta_{t+1})\right] \tag{3.13}$$

The first-order condition for (3.13) is

$$\lambda\theta_t + a_7 \beta E_t V_{\theta_t}(z_{t+1}) = 0 \tag{3.14}$$

By the Envelope Theorem $V_\theta = V_z$, we have

$$V_z(z_t) = z_t + a_6 \beta E_t V_\theta (z_{t+1}) \tag{3.15}$$

By multiplying (3.15) by a_7 / a_6 , substituting in (3.14), and taking expectations relative to the one-step ahead equation, one obtains

$$E_t V_\theta (z_{t+1}) = a_6 z_t + a_7 \ \theta_t - \frac{a_6 \lambda}{a_7} E_t(\theta_{t+1}) \tag{3.16}$$

By inserting (3.16) into (3.14), we get

$$\theta_t = -\frac{a_6 a_7 \beta}{\lambda + a_7{}^2 \beta} z_t + \frac{a_6 \beta \lambda}{\lambda + a_7{}^2 \beta} E_t(\theta_{t+1}) \tag{3.17}$$

When the policy is established at instant t, z_t is the state variable, and we will look for policy rules of the form $\Theta_t = Xz_t$, where X is to be determined9F[6]. Since

$$E_t(\theta_{t+1}) = XE_t(z_{t+1}) = X(a_6 + a_7 X)z_t \tag{3.18}$$

we see that X is the solution to the second-degree equation

$$\lambda \beta a_6 a_7 X^2 + (-\lambda + \beta a_6^2 \lambda - a_7^2 \beta)X - a_6 a_7 \beta = 0 \tag{3.19}$$

It follows that

$$X = \frac{(\lambda - \beta a_6^2 \lambda + a_7^2 \beta) \pm \sqrt{(-\lambda + \beta a_6^2 \lambda - a_7^2 \beta)^2 + 4(a_6^2 a_7^2 \beta^2 \lambda)}}{2\beta a_6 a_7 \lambda} \tag{3.20}$$

After some algebra, we see that the product of the roots (X_1 and X_2) is

$$X_1 X_2 = -\frac{1}{\lambda} < 0 \tag{3.21}$$

6 See Walsh (2003:508-511).

and therefore the roots have opposite signs. Any root satisfying the stability condition may be used[7] to define the optimal rule. In our empirical exercise only the negative root X_2 (say) satisfies this condition. Substituting the negative root in (3.20) into $\theta_t = X_2 z_t$ leads to

$$\theta_t = \frac{(\lambda - \beta\, a_6^2 \lambda + a_7^{\,2} \beta) - \sqrt{(-\lambda + \beta a_6^2 \lambda - a_7^{\,2} \beta)^2 + 4(a_6^{\,2} a_7^2 \beta^{\,2} \lambda)}}{2\beta a_6 a_7 \lambda} z_t$$

(3.22)

By inserting the (3.6) and (3.7) into $\theta_t = X_2 z_t$, we derive the following optimal rule for the nominal interest rate,

$$R_t = \frac{a_7 X_2 - \alpha_1}{a_3} y_t + \frac{a_6 X_2 + a_3 a_6}{a_3} \pi_t - \frac{a_4}{a_3} \mathit{fl}_t - \frac{a_5}{a_3} e_t + \frac{a_8 X_2}{a_3} \Delta q_t$$

(3.23)

We note that

$$\frac{\partial R_t}{\partial y_t} > 0; \frac{\partial R_t}{\partial \pi_t} > 0; \frac{\partial R_t}{\partial \mathit{fl}_t} > 0; \frac{\partial R_t}{\partial e_t} > 0; \frac{\partial R_t}{\partial \Delta q_t}$$

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[7] Since $z_{t+1} = a_6 z_t + a_7 \theta_t + a_8 \Delta q_{t+1} + a_6 \eta_{t+1} + a_7 u_{t+1}$

and $\Theta_t = X z_t$ then

$z_{t+1} = a_6 z_t + a_7 X z_t + a_8 \Delta q_{t+1} + a_6 \eta_{t+1} + a_7 u_{t+1} = (a_6 + a_7 X) z_t + a_8 \Delta q_{t+1} + a_6 \eta_{t+1} + a_7 u_{t+1}$

The stability of the inflation process requires $|a_6 + a_7 X| < 1$.

3 EMPIRICAL RESULTS: IS/PHILLIPS EQUATIONS

In our empirical work, we use quarterly data from 1996:I to 2007:I. All variables are in natural logs. As a proxy for real output gap (y), we compute the difference between GDP and the same GDP series smoothed by the Hodrick-Prescot filter. The inflation rate (π) is measured by IPCA, the Brazilian consumer price index used by the Central Bank to target inflation. The nominal interest rate, (R), is SELIC, which is the primary interest rate in Brazil. We deflated the SELIC by IPCA to obtain the real interest rate, r. We consider the nominal fiscal deficit (fd / GDP), i.e., the change of the ratio public debt/GDP as the relevant fiscal variable. The effective real exchange rate (e) is a proxy for the real exchange rate. Changes in the nominal exchange rate are measured by changes in the effective nominal exchange rate. We use a dummy variable (D) to capture the effect of the period 1996-1998 in which the exchange rate was administered. The source of data is the Brazilian Central Bank.

Table 4.1 shows a unit root test including a constant for variables in level and no constant otherwise. The tests reject the unit root hypothesis for fd / GDP , Δ real exchange rate, Δ nominal exchange rate, output gap, inflation rate and the real interest rate at the 5% level. Further inspections of the correlograms of these series suggest stationarity.

Table 4.1 – Unit Root Tests

Variables	ADF Test	P - Value	Phillips Perron Test	P – Value
Δ (Debt/GDP)	-2.604	0.011	-6.824	<0.001
Δ Real exchange rate	-5.468	<0.001	-5.376	<0.001
Δ Nominal exchange rate	-5.118	<0.001	-5.118	<0.001
Output gap	-3.774	<0.001	-3.816	<0.001
Inflation rate	-4.301	0.001	-4.361	0.001
Real interest rate	-3.179	0.028	-3.080	0.036

Since the nominal interest rate is not a stationary process, we opt to use the real interest rate such that the expression (3.1) becomes

$$y_t = c_1 + a_1 y_{t-1} + a_2 y_{t-2} + a_3 r_{t-1} + a_4 \Delta d_{t-1} + a_5 \Delta e_{t-1} + u_t \qquad (4.1)$$

We also use Δe in (3.1) since e may not be stationary. We follow Freitas and Muinhos (2002). They use Δe in the specification of the IS curve. The increase in the change in the real exchange rate implies domestic currency depreciation

The Phillips curve is

$$\pi_t = c_2 + c_3 D + a_6 \pi_{t-1} + a_7 y_{t-1} + a_8 \Delta q_{t-1} + \eta_t \qquad (4.2)$$

This system defined by Equations (4.1) and (4.2) is estimated by GMM method. Tables 4.2 and 4.3 show the parameter estimates of Equations (4.1) and (4.2), respectively.

Table 4.2 - Results of the Estimation – Method GMM

$$y_t = c_0 + a_1 y_{t-1} + a_2 y_{t-2} + a_3 r_{t-1} + a_4 fd_{t-1} + a_5 \Delta e_{t-1} + u_t$$

Variable	Coefficient	t-Statistic	p-value
Constant	0.002	3.338	0.002
Output gap (-1)	0.892	20.164	<0.001
Output gap (-2)	-0.452	-11.463	<0.001
Real interest rate (-1)	-0.256	-5.141	<0.001
Δ Debt /GDP) (-1)	0.041	7.067	<0.001
Δ (Real exchange rate) (-1)	-0.017	-7.573	< 0.001
R^2	0.487		

Instruments: R (-1 to -2), y (-1 to -3), Δe (-1 to -4), Δq (-1 to -4), π (-1 to -3), Dummy, Constant.

Table 4.3 - Results of the Estimation – Method GMM

$$\pi_t = c_2 + c_3 D + a_6 \pi_{t-1} + a_7 y_{t-1} + a_8 \Delta q_{t-1} + \eta_t$$

Variable	Coefficient	t-Statistic	p-value
Constant	0.350	20.535	<0.001
Dummy	-0.245	-15.024	<0.001
Output gap (-1)	0.884	2.252	0.027
Change in nominal exchange rate (-1)	0.795	14.758	<0.001
Inflation rate (-1)	0.195	6.201	<0.001
R^2	0.342		

Instruments: R (-1 to -2), y (-1 to -3), Δe (-1 to -4), Δq (-1 to -4), π (-1 to -3), Dummy, Constant.

The J-statistic is 0.28 with a p-value of 0.75 and, therefore, there is no evidence to reject the model specification. All parameters in (4.1) and (4.2) are highly significant. All estimates show the expected signs with the exception of output gap (-2) and the change of the real exchange rate. Indeed, the output gap lags in Equation (4.1) show opposite signs. This contrast is necessary to control the seasonality pattern of inertia and dynamics strongly present in time series of the output. The negative parameter for the change in the real exchange rate is explained by Blanchard (2004)[8].

The effect of interest rate on inflation is indirect. A quarterly 1% increase in real interest rate will negatively affect the output gap in 0.256%. Given that a 1% decrease in output gap reduces inflation by 0.884%, the final effect of a 1% increase in the interest rate will cause a 0.226% quarterly decrease in the inflation rate. In the same vein, the effect of fiscal deficit on inflation is also indirect. A 1% quarterly decrease in the nominal deficit/GDP ratio will decrease output gap by 0.041%. A 1% decrease in output gap reduces inflation by 0.884%. The final effect of a 1% decrease in fiscal deficit will be a reduction of 0.036% in the inflation rate.

4 THE OPTIMAL RULE

We assume $\beta = 0{,}98$[9] and $\lambda = 1$ in the derivation of the optimal rule. These parameters are the intertemporal discount

[8] We discuss this point in the next section.
[9] We follow Lima and Issler (2003).

factor and the relative weight of output gap in the loss function, respectively. Substituting the values for β and λ and the parameter estimates shown in Tables 4.2 and 4.3 into (3.23) leads to

$$R_t = 0.335 y_t + 0.269 \pi_t + 0.160 fd_t - 0.066 \Delta e_t + 0.301 \Delta q_t \quad (5.1)$$

where $X_2 = -0.097$. According to the optimal rule, the nominal interest rate increases less than 1% for each 1% increase for any given 1% relative change in any of the right-hand side variables in (5.1), *ceteris paribus*, except the change of real exchange rate. Blanchard (2004) considers the efficacy of the Brazilian monetary policy in 2002 and 2003. This author argues based on a standard proposition that an increase in the real interest rate makes domestic government more attractive and leads to a real appreciation. However, if the increase in the real interest rate also increase the probability of default on the debt, the effect may be instead to make domestic government debt less attractive, and to leads to a real depreciation. He points to the perverse effects of the monetary policy undertaken by the Central Bank with the pursue of the inflation targeting regime in an environment of fiscal dominance regime. It seems to be the case here.

The optimal rule policy ensures a nominal interest rate that will act to lower inflation. We note that the nominal interest rate reacts to changes in public debt and it also suggests a fiscal dominance regime.

Another important issue in the context of the theoretical model of Section II is that the stability condition $|a_6 + a_7 X_2| = 0.109 < 1$ holds.

5 CONCLUSIONS

This article aimed to assess Brazilian economic policies in a context of fiscal disequilibrium from 1996: I to 2007: I. The empirical evidence is that the fiscal deficit affects the output gap directly and, consequently, affects the inflation rate indirectly. Hence, the optimal monetary rule calls for changes in the nominal interest rate in response to changes in the fiscal deficit. These results indicate a fiscal dominance regime. Furthermore, there are evidences that suggest a perverse effect of the interest rate on the exchange rate and on inflation.

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