

PREDICTION OF RESERVOIR FLOW RATE OF DEZ DAM BY THE PROBABILITY MATRIX METHOD

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Abstract:

The data collected from the operation of existing storage reservoirs, could offer valuable information for the better allocation and management of fresh water rates for future use to mitigation droughts effect. In this paper the long-term Dez reservoir (IRAN) water rate prediction is presented using probability matrix method. Data is analyzed to find the probability matrix of water rates in Dez reservoir based on the previous history of annual water entrance during the past and present years(40 years). The algorithm developed covers both, the overflow and non-overflow conditions in the reservoir. Result of this study shows that in non-overflow conditions the most exigency case is equal to 75%. This means that, if the reservoir is empty (the stored water is less than 100 MCM) this year, it would be also empty by 75% next year. The stored water in the reservoir would be less than 300 MCM by 85% next year if the reservoir is empty this year. This percentage decreases to 70% next year if the water of reservoir is less than 300 MCM this year. The percentage also decreases to 5% next year if the reservoir is full this year. In overflow conditions the most exigency case is equal to 75% again. The reservoir volume would be less than 150 MCM by 90% next year, if it is empty this year. This percentage decreases to 70% if its water volume is less than 300 MCM and 55% if the water volume is less than 500 MCM this year. Result shows that too, if the probability matrix of water rates to a reservoir is multiplied by itself repeatedly; it converges to a constant probability matrix, which could be used to predict the long-term water rate of the reservoir. In other words, the probability matrix of series of water rates is changed to a steady probability matrix in the course of time, which could reflect the hydrological behavior of the watershed and could be easily used for the long-term prediction of water storage in the down stream reservoirs.

Keywords: Drought risk management; water resources; probability matrix.

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INTRODUCTION

As water resources become limited while population is growing in the arid and semi-arid regions, there will always be a need to develop more water resources. One of the most promising avenues for addressing water shortfalls in the recent years is water management and incentive policy reform to enhance the efficiency of existing water use. Efficiency of water use in the watersheds depend upon improvements in water-saving technologies, governing water allocation, water rights and water quality (Rosegrant *et al.*, 2002).

Drought and shortage of fresh water is currently an important limitation of water resources development. While in last decades a great deal of effort is increasingly diverted towards the use of poor quality water, the issues related to the storage and management of fresh water have not yet been properly addressed. The regulation of stochastically fluctuating flows of a natural stream by the conservation storage of a reservoir is a classical problem, known for thousands of years.

Many studies have been performed with many different methods and different results for the relationship between storage capacity and target draft, which is defined as firm yield for a failure-free operation over a fixed service period. After Rippl (1883) originated the classical mass curve analysis, many innovative and astonishingly different approaches were put forth by many notable researchers, a few of whom are: Hazen (1914), Hurst (1951), Moran (1959), Fiering (1963; 1965), Hardison (1966), U.S. Army Corps of Engineers (1975), McMahon and Mein (1978), Klemes (1979), and Vogel & Stedinger (1987). Various definitions of yield and risk or reliability of obtaining this yield from a reservoir have been suggested, which mainly depend on stream flow characteristics, storage capacity, evaporation losses, and reservoir service life. The data collected from the operation of the existing storage reservoirs, could offer valuable information for the better allocation and management of water rates for the future use.

Others have successfully used the probability matrix method for different purposes. Gupton (1996) employed the matrix probability method through the multiplication of matrix by itself to obtain a two-year ratings transition matrix, which provides the desired default probabilities. Senior and Green (2004) used the probability matrix to generate a risk index matrix in order to improve the development of a decision-support scheme. Rafecas *et al.* (2004) used the probability matrix method, by sorting the simulated data into a matrix, to calculate probability system by means of Monte-Carlo simulations. Rosenberg & Werman (2004) have also used the probability distribution matrix method to extract and represent the motion displacement between

two images. Saei *et al.* (2009) used the improved SPA methods and probability matrix method and provided ultimately appropriate control curves to obtain various combinations of the quantity of demand and functional indices. The obtained results display a good accordance between the results obtained by SPA and probability matrix methods. There occurred the maximum difference between the methods in minimum sustainability and the minimum difference in maximum sustainability. Rahimi & Montasery (2011) applied Behavior Analysis and SPA method for analysis of store size of Barandvz and Nazlvchi dams. Extraction results were reviewed and completed by using probability matrix method. In this paper the long-term Dez reservoir (IRAN) flow data is analyzed to define an algorithm based on the 40 years historical annual inflow data in Dez reservoir. Investigation has been conducted for both, the overflow and non-overflow conditions in reservoirs.

MATERIALS AND METHODS

Study area

The Dez Dam is a large hydroelectric dam built in Iran in 1963 by an Italian consortium. The dam is on the Dez River, the closest city being Dezful the only way to access the dam in the Northwestern province of Khuzestan. It is 214 m high, making it one of the highest in the world, and has a reservoir capacity of 3,340 million cubic meters. At the time of construction the Dez Dam was Iran's biggest development project.

Data

The annual inflow rate during the past forty years is tabulated at **Table 1**. The data in **Table 1** has been categorized in eight groups according to the flow rates ranging from 101 to 520 million cubic meters (MCM). **Table 2** shows the eight groups and the related flow rates.

Reservoir specification and policy

Suppose the capacity of a storage dam is m unit, and the rate of inflow to the reservoir in the n th year is y_n . Therefore, $y_n = 1$ indicates that the ratio of inflow to the reservoir would be $1/m$ in the n_{th} year of operation. Likewise $y_n = 2$ shows that the ratio of inflow to the reservoir would be $2/m$ in the n_{th} year of operation. Thus, the ratio $y_n = m$ denotes to a full reservoir in the n th year of the operation. The storage or release of water depends on the rate of domestic, industrial, agricultural and environmental demand from the reservoir.

Table 1. The annually inflow rate during past forty years

| | | | | | | | | | | |
|-----------------|-------------|-------------|-------------|-------------|-------------|-------------|-------------|-------------|-------------|-------------|
| NO. | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| Year | 1966 | 1967 | 1968 | 1969 | 1970 | 1971 | 1972 | 1973 | 1974 | 1975 |
| Flow-rate (MCM) | 224.05 | 161.24 | 150.28 | 338.46 | 341.34 | 138.58 | 279.80 | 357.89 | 189.07 | 265.48 |
| NO. | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 |
| Year | 1976 | 1977 | 1978 | 1979 | 1980 | 1981 | 1982 | 1983 | 1984 | 1985 |
| Flow-rate (MCM) | 269.21 | 302.02 | 248.94 | 230.77 | 267.83 | 374.38 | 259.48 | 245.09 | 232.80 | 198.35 |
| NO. | 21 | 22 | 23 | 24 | 25 | 26 | 27 | 28 | 29 | 30 |
| Year | 1986 | 1987 | 1988 | 1989 | 1990 | 1991 | 1992 | 1993 | 1994 | 1995 |
| Flow-rate (MCM) | 205.64 | 327.70 | 385.07 | 253.20 | 249.81 | 199.59 | 226.25 | 447.87 | 343.68 | 233.55 |
| NO. | 31 | 32 | 33 | 34 | 35 | 36 | 37 | 38 | 39 | 40 |
| Year | 1996 | 1997 | 1998 | 1999 | 2000 | 2001 | 2002 | 2003 | 2004 | 2005 |
| Flow-rate (MCM) | 235.88 | 278.35 | 267.42 | 121.82 | 169.84 | 214.48 | 253.41 | 295.65 | 313.49 | 291.18 |

Table 2. The category of annual inflow rates

| Flow-rate (MCM) | Class | Frequency | Percentage |
|-----------------|-------|-----------|------------|
| < 100 | 0 | — | — |
| 101-150 | 1 | 2 | 5.0 |
| 151-200 | 2 | 6 | 15.0 |
| 201-250 | 3 | 11 | 27.5 |
| 251-300 | 4 | 11 | 27.5 |
| 301-350 | 5 | 6 | 15.0 |
| 351-400 | 6 | 3 | 7.5 |
| 401-450 | 7 | 1 | 2.5 |
| > 450 | 8 | — | — |

Ordaining probability matrix

Considering the annul demand (including evaporation and leakage) amounts to 800 MCM, if the water entrance rate to the reservoir is less, all water is released in the river for down stream use. However, if the water entrance rate to the reservoir is more than 800 MCM, the 800 MCM would be released and the rest is stored in the reservoir. For example, an annual inflow of 1000 MCM water to a reservoir would result in 200 MCM storage in the reservoir, and 800 MCM release in the river for the utilization.

Now, if x is the volume of the stored water in the reservoir at the end of each year, then x_n and $x_{(n+1)}$ refer to the volume of stored water in the reservoir at the end of n th and $(n+1)$ th year, respectively. In other words, $x_{(n+1)}$ is the volume of stored water at the end of n th year (x_n) plus the volume of inflow in the $(n+1)$ th year ($y_{(n+1)}$) minus the volume of released water in the $(n+1)$ th year. In order to define the algorithm of stored water P from the n th to $(n+1)$ th year the following equation will hold:

$$P_{ij} = P\{x_{(n+1)} = j | x_n = i\} \tag{1}$$

where i and j are the units of water in the reservoir at the end of n th and $(n+1)$ th year respectively. Using **Eq. (1)**, the probability of various conditions may be calculated. If the storage capacity of a full and empty reservoir is assumed as 4 and 0 units, respectively, the probability of other cases such as P_{00} , P_{10} , P_{11} , P_{20} , P_{21} and P_{44} may be calculated as follows:

- a. P_{00} : The possibility of reservoir being empty at the end of this year ($i = 0$) and next year ($j = 0$);

$$P_{00} = P\{x_{(n+1)} = 0 | x_n = 0\}$$

$$P_{00} = P[y_{(n+1)} \leq 4]$$

$$P_{00} = P[y_{(n+1)} = 0] + P[y_{(n+1)} = 1] + P[y_{(n+1)} = 2] + P[y_{(n+1)} = 3] + P[y_{(n+1)} = 4]$$

$$P_{00} = 0\% + 5\% + 25\% + 25\% + 15\% = 70\%$$

- b. P_{10} : The possibility of reservoir with one unit stored water ($i = 1$), being empty next year ($j = 0$);

$$P_{10} = P\{x_{(n+1)} = 0 | x_n = 1\}$$

$$P_{10} = P[y_{(n+1)} \leq 3]$$

$$P_{10} = P[y_{(n+1)} = 0] + P[y_{(n+1)} = 1] + P[y_{(n+1)} = 2] + P[y_{(n+1)} = 3]$$

$$P_{10} = 0\% + 5\% + 25\% + 25\% = 55\%$$

- c. P_{11} : The possibility of reservoir with one unit stored water ($i=1$), having one unit stored water next year too ($j=1$);

$$P_{11} = P\{x_{(n+1)} = 1 | x_n = 1\}$$

$$P_{11} = P[y_{(n+1)} = 4] = 15\%$$

- d. P_{20} : The possibility of reservoir with two units stored water ($i = 2$), being empty next year ($j = 0$);

$$P_{20} = P\{x_{(n+1)} = 0 \mid x_{(n)} = 2\}$$

$$P_{20} = P[y_{(n+1)} \leq 2]$$

$$P[y_{(n+1)} = 0] + P[y_{(n+1)} = 1] + P[y_{(n+1)} = 2] = 0\% + 5\% + 25\% = 30\%$$

e. P_{21} : The possibility of reservoir with two units stored water ($i = 2$), having one unit stored water next year ($j = 1$);

$$P_{21} = P\{x_{(n+1)} = 1 \mid x_{(n)} = 2\}$$

$$P_{21} = P[y_{(n+1)} = 3] = 25\%$$

f. P_{44} : The possibility of reservoir being full this year ($i = 4$), and next year too ($j = 4$);

$$P_{44} = P\{x_{(n+1)} = 4 \mid x_{(n)} = 4\}$$

$$P_{44} = P[y_{(n+1)} \geq 4]$$

$$P_{44} = P[y_{(n+1)} = 4] + P[y_{(n+1)} = 5] + P[y_{(n+1)} = 6] + P[y_{(n+1)} > 6]$$

$$P_{44} = 15\% + 15\% + 10\% + 5\% = 45\%$$

g. Considering the other possibilities, the probability matrix of the water rates in the reservoir P_{ij} is:

$$\begin{pmatrix} P_{00} & P_{01} & P_{02} & P_{03} & P_{04} \\ P_{10} & P_{11} & P_{12} & P_{13} & P_{14} \\ P_{20} & P_{21} & P_{22} & P_{23} & P_{24} \\ P_{30} & P_{31} & P_{32} & P_{33} & P_{34} \\ P_{40} & P_{41} & P_{42} & P_{43} & P_{44} \end{pmatrix} = \begin{pmatrix} 75 & 15 & 7.5 & 2.5 & 0 \\ 47.5 & 27.5 & 15 & 7.5 & 2.5 \\ 20 & 27.5 & 27.5 & 15 & 10 \\ 5 & 15 & 27.5 & 27.5 & 25 \\ 0 & 5 & 15 & 27.5 & 52.5 \end{pmatrix} [1]$$

Many conclusions may be drawn from matrix [1], for example:

- a. $P_{12} = 15\%$. In the matrix means that, if the reservoir water rate is between 100-150 *MCM* (1 unit) this year and between 150-200 *MCM* (2 units) next year, its possibility is equal to 15%.
- b. $P_{44} = 52.5\%$. In the matrix means that if the reservoir is full (water volume is more than 500 *MCM*) this year, it would be full by 52.5% probability next year too.
- c. P_{00} is the most exigency case, with a probability equal to 75%. This means that, if the reservoir is empty (the stored water is less than 100 *MCM*) this year, it would be also empty by 75% next year.
- d. The stored water in the reservoir would be less than 150 *MCM* by 90% ($75\% + 15\%$) next year if the reservoir is empty this year. This percentage decreases to 63% ($47.5\% + 15.5\%$) next year if the water of reservoir is less than 150 *MCM* this year. The percentage also decreases to 5% next year if the reservoir is full this year.

Permanent probability matrix

Now, if matrix [1] is multiplied by itself repeatedly, the long-term probability matrix is found. A permanent condition is resulted by nineteenth year if the probability matrix is accounted for the future years. In this case the nineteenth year probability matrix is:

$$\begin{pmatrix} 75 & 15 & 7.5 & 2.5 & 0 \\ 47.5 & 27.5 & 15 & 7.5 & 2.5 \\ 20 & 27.5 & 27.5 & 15 & 10 \\ 5 & 15 & 27.5 & 27.5 & 25 \\ 0 & 5 & 15 & 27.5 & 52.5 \end{pmatrix}^{19} = \begin{pmatrix} 48 & 18 & 14 & 10 & 9 \\ 48 & 18 & 14 & 10 & 9 \\ 48 & 18 & 14 & 10 & 9 \\ 48 & 18 & 14 & 10 & 9 \\ 48 & 18 & 14 & 10 & 9 \end{pmatrix} [2]$$

Many conclusions are obtained from matrix [2], for example:

- a. It is seen that in each row of matrix [2] the percentages are equal. This means that, the present water volume of reservoir does not have any effect on the reservoir's water rate ten years later.
- b. The most exigency case is the first column of the matrix, with a probability equal to 48%. This means that, the reservoir would be empty (less than 100 *MCM*) by 48% nineteenth years later.
- c. The water volume in the reservoir would be less than 150 *MCM* by 66% ($48\% + 18\%$), 200 *MCM* by 80% ($48\% + 18\% + 14\%$), and 250 *MCM* by 90% ($48\% + 18\% + 14\% + 10\%$) ten years later.
- d. The water volume in the reservoir would be more than 250 *MCM* by 9% nineteenth years later.

Ordaining probability matrix considering overflow

What was discussed before denotes the probability matrix of non-overflow condition. However, if the reservoir is supposedly full, we may have over flow when the inflow rate increases. To account the real rates, the probability matrix will be as follows:

$$\begin{pmatrix} P_{00} & P_{01} & P_{02} & P_{03} & P_{04} \\ P_{10} & P_{11} & P_{12} & P_{13} & P_{14} \\ P_{20} & P_{21} & P_{22} & P_{23} & P_{24} \\ P_{30} & P_{31} & P_{32} & P_{33} & P_{34} \\ P_{40} & P_{41} & P_{42} & P_{43} & P_{44} \end{pmatrix} = \begin{pmatrix} 75 & 15 & 7.5 & 2.5 & 0 \\ 47.5 & 27.5 & 15 & 10 & 0 \\ 20 & 27.5 & 27.5 & 25 & 0 \\ 5 & 15 & 40 & 40 & 0 \\ 0 & 0 & 5 & 95 & 0 \end{pmatrix} [3]$$

This matrix (3) shows that, for example:

- a. P_{00} is the most exigency case, with a probability equal to 75% again. (Compare it with matrix [1]).
- b. The last column of matrix [3] is zero. It means that the reservoir will not be full next year because of overflow. In other words, the possibility of being full has been completely decreased.

- c. The reservoir volume would be less than 150 *MCM* by 90% (75% + 15%) next year, if it is empty this year. This percentage decreases to 75% (47.5% + 27.5%) if its water volume is less than 150 *MCM* and 47.5% (20% + 27.5%) if the water volume is less than 200 *MCM* this year.

Permanent probability matrix considering overflow

Now, if matrix [2] is multiplied by itself repeatedly, the long-term probability matrix is found considering overflow. In this case the eleventh year probability matrix is:

$$\begin{pmatrix} 75 & 15 & 7.5 & 2.5 & 0 \\ 47.5 & 27.5 & 15 & 10 & 0 \\ 20 & 27.5 & 27.5 & 25 & 0 \\ 5 & 15 & 40 & 40 & 0 \\ 0 & 0 & 5 & 95 & 0 \end{pmatrix}^{11} = \begin{pmatrix} 52 & 19 & 16 & 12 & 0 \\ 52 & 19 & 16 & 12 & 0 \\ 52 & 19 & 16 & 12 & 0 \\ 52 & 19 & 16 & 12 & 0 \\ 52 & 19 & 16 & 12 & 0 \end{pmatrix} \quad [4]$$

From this matrix (4) we could learn that, for example:

- The most exigency case is the first column, with a probability equal to 52% which is greater than the first column of matrix [2]. It means that, the reservoir would be empty by 52% at eleventh year due to the overflow.
- The reservoir volume will be less than 150 *MCM* by 71% (52%+19%) in the tenth year.
- The last column of matrix [4] is also zero. (Compare it with matrix [2]). It means that the reservoir will not be full in the eleventh year because of overflow. In other words, the possibility of being full is zero ten years later.

CONCLUSION

A water management technique has been developed and presented based on the long-term probability matrix of water volume in the storage reservoir for both, with and without reservoir spill. This method uses the past long-term reservoir volumes to predict its future behavior. Data has been analyzed to find the algorithm for the probability matrix of water rates in Dez reservoir using the 40 years historical annual inflow data. The results have shown that by setting up the long-term probability matrix, the future behavior of the reservoir could be predicted. Since in the regulated rivers the down stream domestic, industrial, agricultural and environmental water needs are highly dependent upon the available

stored water, therefore this technique could highly improve the operation of a storage reservoir by predicting available water in future years.

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