

## THE EFFECT OF THE REYNOLDS NUMBER OF AIR FLOW TO THE PARTICLE COLLECTION EFFICIENCY OF A FIBROUS FILTER MEDIUM WITH CYLINDRICAL SECTION

George P. Kouropoulos\*

*Department of Energy Technology, Faculty of Technological Applications, Technological Educational Institute of Athens, Greece*

Received 7 November 2013; received in revised form 12 December 2013; accepted 18 January 2014

**Abstract:**

At this study an attempt for the theoretical approach of the Reynolds number effect of air flow to the particle collection efficiency of a fibrous filter with cylindrical section will be made. Initially, a report of the air filtration models to fibrous filter media will be presented along with an explanation of both the parameters and the physical quantities which govern the air filtration process. Furthermore, the resulting equation from the mathematical model will be applied to a real filter medium and the characteristic curves of filter efficiency will be drawn. The change of a filter medium efficiency with regard to the Reynolds number of air flow that passes through the filter, derived from the curves, will be studied. The general conclusion that we have is that as the Reynolds number of filtered air increases, the collection efficiency of the filter decreases.

**Keywords:**

Fibrous filter; Efficiency of filter media; Reynolds number of filter media; Air filtration; Air sterilization;

© 2014 Journal of Urban and Environmental Engineering (JUEE). All rights reserved.

---

\* Correspondence to: George P. Kouropoulos. E-mail: [etmecheng@gmail.com](mailto:etmecheng@gmail.com)

## INTRODUCTION

Fibrous filters media are a filter category which is used for the filtration of airborne pollution particles found in a wide range of applications such as heavy industry and hospitals in order to sterilize the air from viruses, bacteria, pathogen microorganisms and pollutants.

The most important information taking into consideration for the choice of the appropriate filter is the efficiency regarding the collection of these particles. The filters' efficiency is known from the characteristic curves and is chosen according to the particles' diameter we want to withhold by using the filter.

In the past, there were scientists that studied the filtration mechanism of airborne particles and analyzed the efficiency of filters in relation to physical quantities such as the particle diameter and the speed of filtered air (Kowalski *et al.*, 1999; Lee and Liu, 1980; Wang *et al.*, 2006). Furthermore, a correlation between the filter efficiency and a product which included the Reynolds number on a graph was formulated (Lee and Liu, 1980).

The efficiency of filter media for the filtration of airborne particles consists of three mechanisms; interception, diffusion and inertial impaction (Wang *et al.*, 2006). Every one of these mechanisms has distinct degree of particle collection efficiency. The sum of these three degrees of efficiency, leads us to the total collection efficiency of the filter media.

During interception, the particles follow the air flow and due to their big size, impinge on the fiber of filter media. During diffusion, the particles due to their small size, execute irregular – unpredictable movement and finally impinge on the fiber of filter media. During the inertial impaction the particles impinge on the fiber of filter and are wedged on it (Fig. 1).

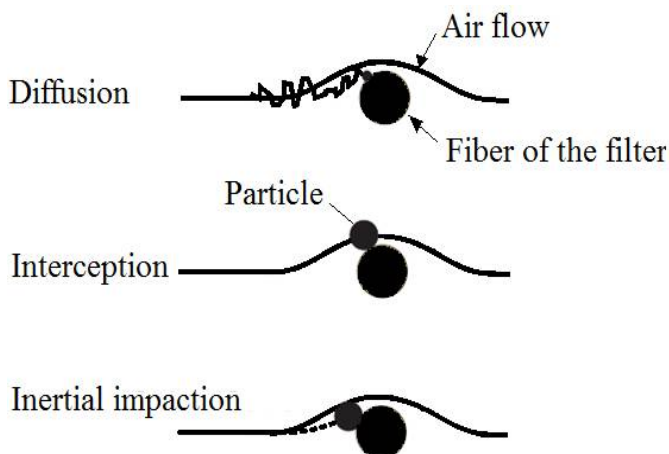


Fig. 1 Virtual representation of the three filtration mechanisms into a filtration medium.

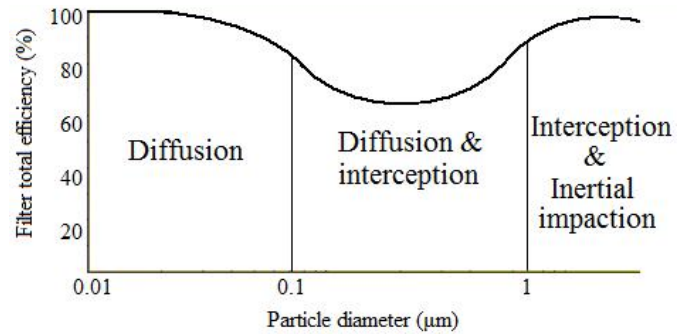


Fig. 2 Characteristic curve of the degree of the collection efficiency in a random filter.

The curve in Fig. 2 shows that the biggest part during the filtration process accounts to the diffusion mechanism. The kind of fluid flow into a closed pipeline is characterized from a dimensionless number which is called Reynolds number. The Reynolds number indicates if the fluid flow is laminar or turbulent.

If we suppose that the fluid passes into a pipeline with internal diameter then the Reynolds number is equal to:

$$Re = \frac{\rho v d}{\mu} = \frac{v d}{\nu} \quad (1)$$

where:

$\rho$  is the density of fluid ( $\text{kg/m}^3$ ).

$v$  is the transit speed of the fluid into pipeline (m/sec).

$d$  is the internal diameter of the cylindrical pipeline (m).

$\mu$  is the absolute viscosity of the fluid ( $\text{kg/m}\cdot\text{sec}$ ).

$\nu$  is the kinematic viscosity of fluid ( $\text{m}^2/\text{sec}$ ).

If the dimensionless Reynolds number varies from 0 to 2000 then the flow is laminar; the fluid molecules are moving parallel with the axes of the duct flow. If the Reynolds number is bigger than 4000 then the flow is turbulent; the fluid molecules execute unpredictable movement. If the Reynolds number varies between  $2000 < Re < 4000$ , then there is a transitional situation for the fluid flow.

## BASIC ASSUMPTIONS OF THE STUDY

We consider that the filter medium, whose change of efficiency will be determined in relation with the Reynolds number of the fluid, has a cylindrical section and is located into a pipeline with cylindrical section, through which air with airborne particles flows. Furthermore, we consider that the section diameter of the cylindrical filter medium is equal with the internal diameter of the pipeline (Fig. 3).

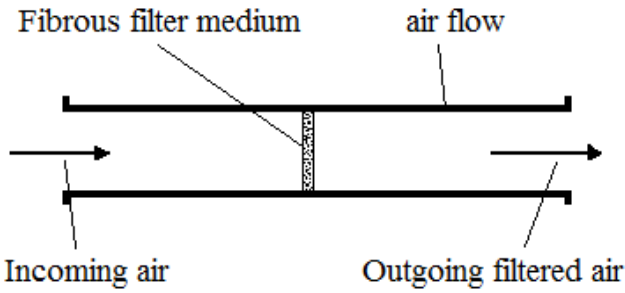


Fig. 3 Virtual representation of a duct air flow with filter insight.

**MATHEMATICAL MODELS OF AIR FILTRATION**

Below all parameters and actual sizes which govern the air filtration with fibrous filter media are analyzed. The total efficiency of a filter medium used for the filtration of air (Wang *et al.*, 2006) is given by the equation:

$$E = 1 - \text{EXP}\left(-\frac{4L\alpha n_{SUM}}{\pi d_f}\right) \tag{2}$$

where:

$\alpha$  is the ratio of the fiber volume  $V_f$  to the total filter medium volume  $V_F$ .

$L$  is the length (thickness) of the filter medium (mm).

$d_f$  is the fiber diameter of the medium (mm).

$n_{SUM}$  is the sum of the collection efficiency for every filtration mechanism (interception, diffusion, impaction).

The particle collection efficiency due to interception  $n_R$  (Kowalski *et al.*, 1999; Wang *et al.*, 2006) is given by the equation:

$$n_R = \frac{(1 - \alpha)N_R^2}{Ku(1 + N_R)} \tag{3}$$

where  $Ku$  is the Kuwabara dimensionless hydrodynamic factor which equals:

$$Ku = \frac{4\alpha - \alpha^2 - 3}{4} - \frac{\ln \alpha}{2} \tag{4}$$

where  $N_R$  is the dimensionless interception parameter; the ratio of particle diameter  $d_d$  ( $\mu\text{m}$ ) to the fiber diameter of the filter medium  $d_f$  ( $\mu\text{m}$ ).

$$N_R = \frac{d_d}{d_f} \tag{5}$$

$\alpha$  is the ratio of fibers' volume  $V_f$  to the total filter medium volume  $V_F$ . If the filter medium has a cylindrical section, then the following equation applies:

$$\alpha = \frac{V_f}{V_F} = \frac{V_f}{L\pi \frac{d_f^2}{4}} = \frac{4V_f}{L\pi d_f^2} \tag{6}$$

where  $d_f$  is the section diameter of the fibrous filter medium.

The particle collection efficiency due to diffusion  $n_D$  (Kowalski *et al.*, 1999) is given by the equation:

$$n_D = 1.6125 \left(\frac{1 - \alpha}{Ku}\right)^{\frac{1}{3}} Pe^{-\frac{2}{3}} \tag{7}$$

where  $Pe$  is the dimensionless Peclet number which equals:

$$Pe = \frac{v d_f \times 10^{-6}}{BkT} \tag{8}$$

where:

$v$  is the transit fluid speed into pipeline (m/sec).

$B$  is the particle mobility (sec/kg).

$k$  is the Boltzmann constant ( $1.3708 \times 10^{-23}$  J/°K).

$T$  is the absolute temperature of air (°K).

The particle mobility  $B$  equals:

$$B = \frac{1 + \left(\frac{0.067}{d_p}\right) (2.492 + 0.84 \text{EXP}(-6.49d_p))}{3 \times 10^{-6} \pi \mu d_p} \tag{9}$$

where  $\mu$  is the absolute viscosity of infiltrated air (kg/m·sec). Replacing Eq. (9) to Eq. (8) we have the final equation for the Peclet number:

$$Pe = \frac{3 \times 10^{-12} \pi \mu v d_f d_p}{kT \left[1 + \left(\frac{0.067}{d_p}\right) (2.492 + 0.84 \text{EXP}(-6.49d_p))\right]} \tag{10}$$

The particle collection efficiency of the filter medium due to inertial impaction (Wang *et al.*, 2006; Yeh & Liu, 1974) equals:

$$n_i = \frac{Stk \times J}{2Ku^2} \tag{11}$$

where  $J$  is a dimensionless parameter, which depends on the  $N_R$  ratio as follows:

$$J = (29.6 - 28\alpha^{0.62})N_R^2 - 27.5N_R^{2.8} \quad N_R < 0.4 \tag{11a}$$

$$J = 2 \quad N_R > 0.4 \tag{11b}$$

where  $Stk$  is the Stokes dimensionless number for the fluid flow in a filter medium, which is given by the following equation:

$$Stk = \frac{\rho d_p^2 v C_D}{18\mu d_f} \tag{12}$$

where  $C_D$  is the dimensionless drag coefficient and  $\rho$ ,  $\mu$ ,  $v$ ,  $d_p^2$ ,  $d_f$  already known quantities. Summing up, the total particle collection efficiency from a fibrous filter medium, results from the replacement of **Eqs (11), (7) and (3) to Eq. (2)**. After the necessary modifications, we have the following equation:

$$E = 1 - \exp \left[ -\frac{4La}{\pi d_f} (n_R + n_D + n_I) \right] \quad (13)$$

**ANALYSIS OF THE FYSICAL QUANTITIES AND PARAMETERS AS A REYNOLDS NUMBER FUNCTION**

**Equation (13)** presents the total efficiency of a fibrous filter medium. We will try, wherever possible, to express the quantities  $a$ ,  $Ku$ ,  $Pe$ ,  $Stk$ ,  $J$ , with regard to the requested Reynolds number. The Reynolds number for fluid flow in a cylinder can be resolved in relation to both the filter diameter  $d_F$  and the flow speed  $v$ .

$$Re = \frac{\rho v d_F}{\mu} \Leftrightarrow d_F = \frac{Re \mu}{\rho v} \Leftrightarrow v = \frac{Re \mu}{\rho d_F} \quad (14)$$

where  $d_F$  is the diameter of the cylindrical medium which is equal to the internal pipeline diameter and  $\rho$ ,  $\mu$ ,  $v$  are known quantities. Replacing **Eq. (14) to Eq. (6)**, the volume ratio of fibers  $V_f$  to the total filter medium volume  $V_F$  in relation to the Reynolds number appears:

$$\alpha = \frac{V_f}{V_F} = \frac{4V_f}{L\pi d_f^2} = \frac{4V_f}{L\pi \left(\frac{Re\mu}{\rho v}\right)^2} = \frac{4V_f \rho^2 v^2}{L\pi \mu^2 Re^2} \quad (15)$$

The fibers filter volume  $V_f$  cannot be expressed as a Reynolds number function. Replacing **Eq. (15) to Eq. (4)** we obtain the dimensionless Kuwabara factor expressed as a Reynolds number function:

$$Ku = \frac{4V_f \rho^2 v^2}{L\pi \mu^2 Re^2} - \frac{4V_f^2 \rho^4 v^4}{L^2 \pi^2 \mu^4 Re^4} - \frac{3}{4} - \frac{1}{2} \ln \left( \frac{4V_f \rho^2 v^2}{L\pi \mu^2 Re^2} \right) \quad (16)$$

Replacing **Eq. (14) to Eq. (10)**, we obtain the dimensionless Peclet number as a Reynolds number function:

$$Pe = \frac{3 \times 10^{-12} \pi d_f d_p \mu^2 Re}{\rho d_F kT \left[ 1 + \left( \frac{0.067}{d_p} \right) (2.492 + 0.84 \text{EXP}(-6.49 d_p)) \right]} \quad (17)$$

Replacing **Eq. (14) to Eq. (12)**, we obtain the dimensionless Stokes number as a Reynolds number function:

$$Stk = \frac{\rho d_p^2 v C_D}{18 \mu d_f} = \frac{\rho d_p^2 Re \mu}{18 \mu d_f} = \frac{d_p^2 Re C_D}{18 d_F d_f} \quad (18)$$

Regarding the drag coefficient  $C_D$ , it can be determined from the Reynolds number, empirically from the curves for fluid flow on solid spherical and cylindrical bodies (Kundu, 1990). In the case of fluid flow during the filtration process in the filter medium, the coefficient factor is given by the following experimental equations in relation with the respective Reynolds number (Tien, 2012).

$$C_D = \frac{24}{Re} \quad 0 < Re < 0.1 \quad (19a)$$

$$C_D = \left( \sqrt{\frac{24}{Re} + 0.5407} \right)^{-2} \quad 0 < Re < 6000 \quad (19b)$$

$$C_D = 0.44 \quad 500 < Re < 100000 \quad (19c)$$

**TECHNICAL CHARACTERISTICS OF FILTER MEDIUM AND CALCULATIONS**

In order to export the curve and monitor the effect of the Reynolds number in the filter efficiency, the characteristics of a real fibrous filter medium are taken into consideration. As a result, the respective curves arising correspond to reality. The following characteristics from a filter medium will be used in this study:

Table 1. Technical characteristics of filter medium		
Characteristic	Symbol	Value
Filter length	$L$	17 mm
Volume ratio	$A$	0.0051
Section	$A$	0.35 m <sup>2</sup>
Speed flow	$Y$	0.035 m/sec
Fiber diameter	$d_f$	0.67 μm

Filter type: High efficiency particular air filter (HEPA), 99.97% maximum efficiency. Filter section: Cylindrical.

From the afore-mentioned characteristics the following elements can be calculated:

Filter diameter  $d_F$ :

$$d_F = \sqrt{\frac{4 \times 0.35 m^2}{\pi}} = 0.668 m = 0.668 \times 10^6 \mu m \quad (20)$$

$$N_R = \frac{d_p}{d_f} = \frac{d_p}{0.67} \quad (27)$$

Filter volume  $V_F$ :

$$V_F = 0.35 m^2 \times 0.017 m = 5.95 \times 10^{15} \mu m^3 \quad (21)$$

Fibers volume  $V_f$ :

$$V_f = \alpha V_F = 0.03 \times 10^{15} \mu m^3 \quad (22)$$

Supposed that the temperature of the air passing through the filter is 20°C (293°K), the density of the passing air is:  $\rho = 1.2 \text{ kg/m}^3$  and the absolute viscosity of the air is:  $\mu = 18.21 \times 10^{-6} \text{ kg/m}\cdot\text{sec}$ .

**INPUT OF VALUES IN THE MATHEMATICAL MODEL**

We input values from Eqs (20–22), and values of Table 1 to Eqs (15–18) and express the quantities as a function of both the Reynolds number and the infiltrated particles diameter  $d_p$ . Replacing  $V_F$ ,  $\rho$ ,  $v$ ,  $\mu$ ,  $L$  to Eq. (15), the factor  $\alpha$  equals:

$$\alpha = \frac{4V_f \rho^2 v^2}{L \pi \mu^2 Re^2} = \frac{83.97}{Re^2} \quad (23)$$

Replacing (23) to (16), the Kuwabara factor equals:

$$Ku = \frac{83.97}{Re^2} - \frac{1762.6}{Re^4} - \frac{1}{2} \ln\left(\frac{83.97}{Re^2}\right) - \frac{3}{4} \quad (24)$$

Replacing  $d_f$ ,  $d_p$ ,  $\mu$ ,  $\rho$ ,  $d_F$ ,  $k$ ,  $T$  to Eq. (17), the Peclet number equals:

$$Pe = \frac{6.5 \times 10^{-7} \times d_p Re}{\left[1 + \left(\frac{0.067}{d_p}\right) (2.492 + 0.84 \text{EXP}(-6.49d_p))\right]} \quad (25)$$

Replacing of  $d_F$  and  $d_f$  to equation, the Stokes number equals (Eq. 18):

$$Stk = \frac{d_p^2 Re C_D}{8.06 \times 10^6} \quad (26)$$

Replacing  $d_f$  to Eq. (5), the dimensionless parameter  $N_R$  equals:

Replacing Eqs (23–27) to the Eq. (13) of total efficiency, we obtain the final function of two variables; the Reynolds number and the particles diameter  $d_p$ , namely  $E(Re, d_p)$ .

At this point, the requested characteristic curve  $E(d_p)$  of the filter medium efficiency can be exported for specific values of Reynolds number which will be chosen in the next chapter.

**CHARACTERISTIC CURVES OF FILTER MEDIUM**

The below – presented curves will be studied in order to monitor the change of the filter medium efficiency for the Reynolds numbers 2000 and 4000. To put it differently, we attempt to monitor what is happening during laminar flow, during turbulent flow, and in the transitional situation. Curves for each filtration mechanism will be exported and finally, the total efficiency will be monitored. We apply Eq. (13) with  $N_R > 0.4$  and we choose the drag coefficient of case (Eq. 19b).

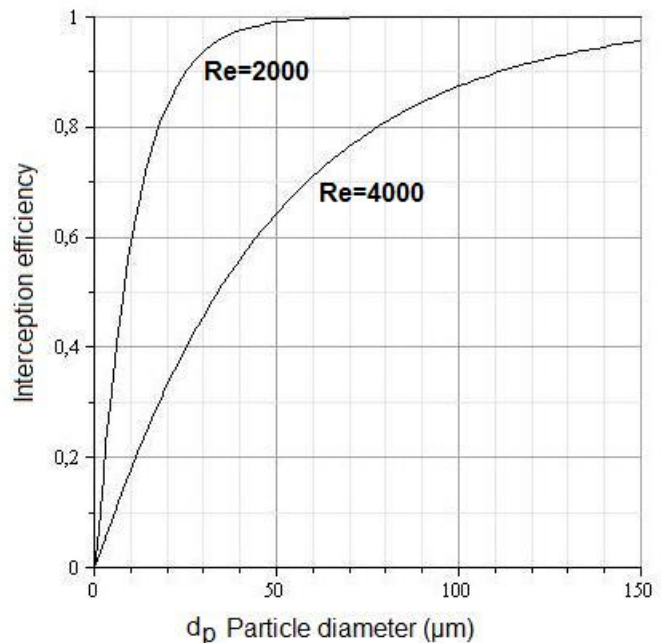
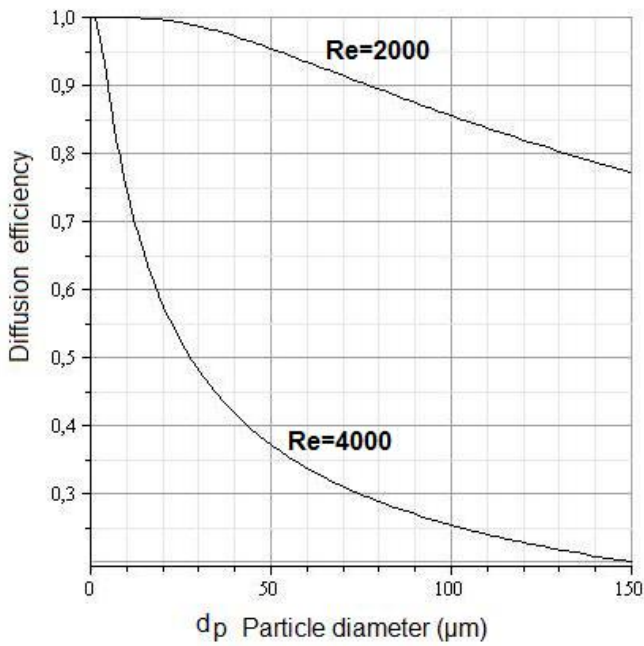
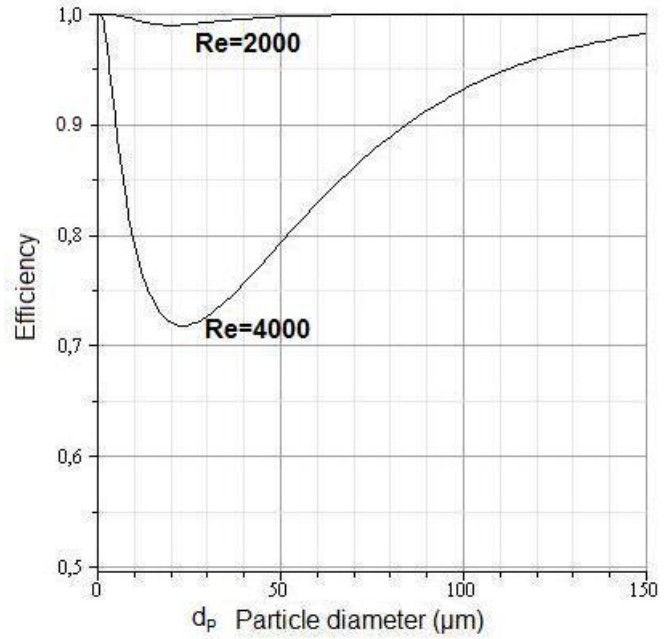


Fig. 4 Change of filter medium efficiency due to the interception mechanism for Re = 2000, Re = 4000.

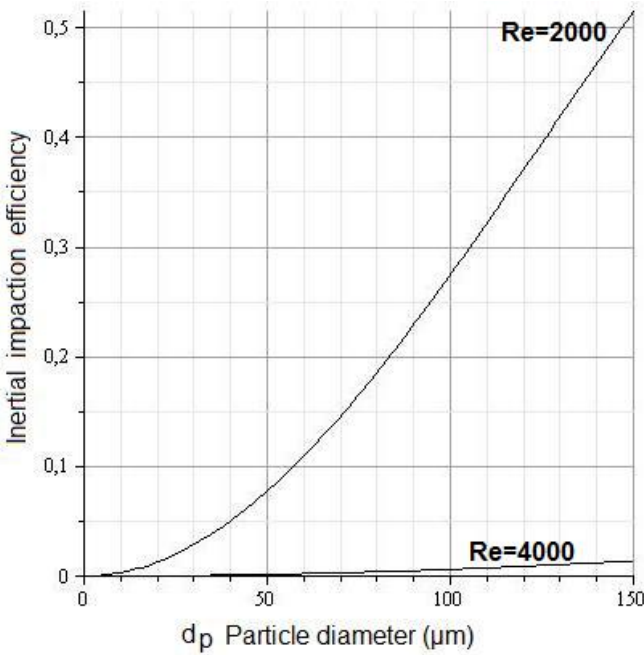




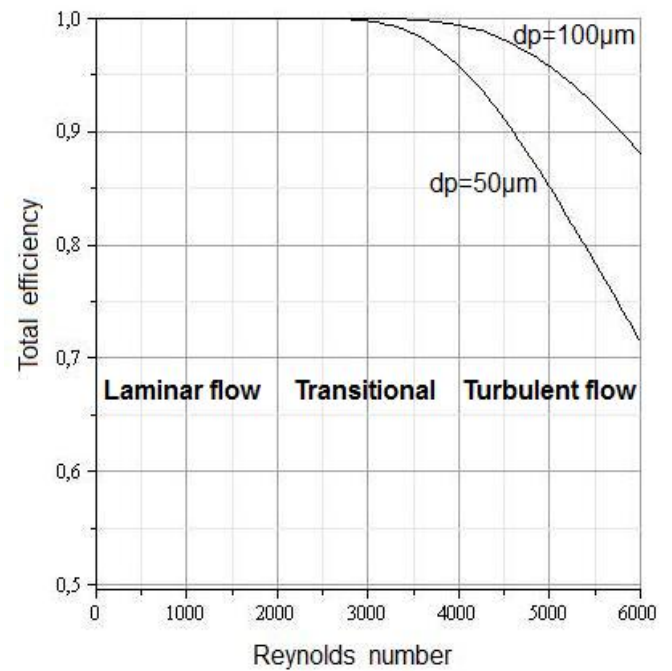
**Fig. 5** Change of filter medium efficiency due to the diffusion mechanism for Re = 2000, Re = 4000



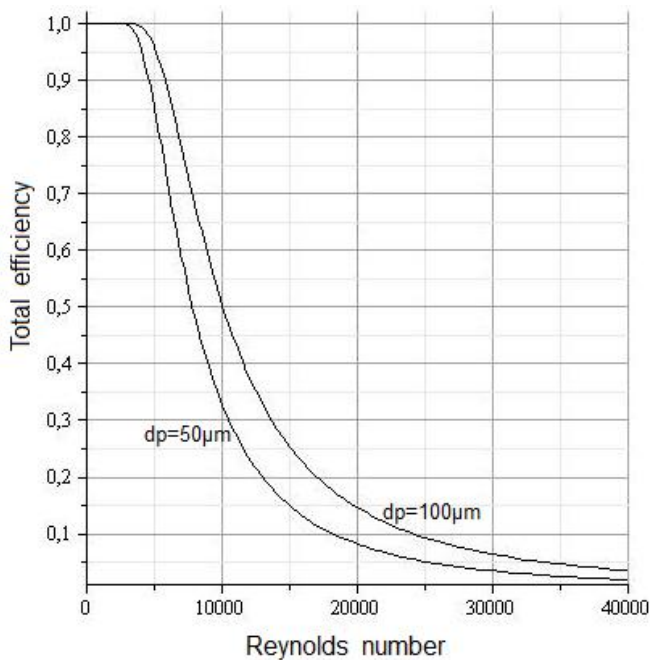
**Fig. 7** Characteristic curves of total particle collection efficiency of the filter medium, for Re = 2000, Re = 4000



**Fig. 6** Change of filter medium efficiency due to the inertial impaction mechanism for Re = 2000, Re = 4000.



**Fig. 8** Change of filter medium total efficiency in relation with the Reynolds number, for  $d_p = 50 \mu\text{m}$ ,  $d_p = 100 \mu\text{m}$  with Reynolds number range of 0–6000.



**Fig. 9** Change of filter medium total efficiency in relation with the Reynolds number, for  $d_p = 50 \mu\text{m}$ ,  $d_p = 100 \mu\text{m}$  with Reynolds number range of 0–40000.

## CONCLUSIONS

From **Figs 4, 5, and 6**, it can be concluded that the efficiency of the filter medium decreases as the Reynolds number of air flow increases for all three filtration mechanisms. During the transition from laminar to turbulent flow the efficiency is decreased for every particle diameter. For small diameter this decrease is approximately  $1 \mu\text{m}$ ; indicating that efficiency of these two flows converges.

Furthermore, in inertial impaction it can be noticed that the increase of the Reynolds number results in a substantial reduction in filter efficiency.

From **Fig. 7**, it can be inferred that the total efficiency of the filter medium decreases as the Reynolds number of the infiltrated air flow increases. More specifically, when the particle diameter  $d_p$  varies from 0 to  $1 \mu\text{m}$ , the efficiency for air flow with  $\text{Re} = 2000$  almost coincides (a marginal deviation is observed) with the respective efficiency for air flow with  $\text{Re} = 4000$ .

When the particle diameter  $d_p$  varies from  $1 \mu\text{m}$  to  $25 \mu\text{m}$ , the efficiency decreases substantially. It reaches the minimum efficiency value which is 72% for  $\text{Re} = 4000$  and 98–99% for  $\text{Re} = 2000$ . We observe a sharp efficiency decrease for air flow with  $\text{Re} = 4000$ .

When the particle diameter  $d_p$  varies from  $25 \mu\text{m}$  to  $150 \mu\text{m}$ , the efficiency for both types of flow; both laminar and turbulent, is recovered and finally, when the particle diameter  $d_p$  surpasses  $150 \mu\text{m}$ , the filter efficiency tends to converge for both types of air flow.

Air filtration with laminar flow, namely  $\text{Re} < 2000$ , results in a high filter efficiency (between  $99\% < E < 99.99\%$ ). Air filtration during the transition situation;  $2000 < \text{Re} < 4000$ , results in an efficiency decrease, while air filtration with turbulent flow, namely  $\text{Re} > 4000$ , results in a large efficiency decrease and more specifically in a substantial decrease in minimum efficiency.

It should be noted that when the Reynolds number doubles, the minimum efficiency is significantly decreased;  $99\% - 72\% = 27\%$ .

From **Fig. 8**, it can be inferred that when air flows have a Reynolds number until 3000, the total filter efficiency remains the same. When air flows have a Reynolds number above 3000, a small decrease begins; a decrease that rises when we transit to turbulent air flow.

Furthermore, it can be concluded that the larger the diameter  $d_p$  of the particle passing from the filter medium, the larger the efficiency for the same Reynolds number. Generally, the findings from **Fig. 7** are confirmed.

From **Fig. 9**, a better and more general picture regarding the change in total efficiency of filter media with regard to the Reynolds number is presented.

When the Reynolds number varies from 3000 to 7500 and the particle diameter  $d_p = 50 \mu\text{m}$  as well as when the Reynolds number varies from 4000 to 12 000 and the particle diameter  $d_p = 100 \mu\text{m}$ , we observe a noteworthy decreasing of filter efficiency, which is approximately linear.

When the Reynolds number varies from 7500 to 35 000 and the particle diameter  $d_p = 50 \mu\text{m}$  as well as when the Reynolds number varies from 12 000 to 35 000 and the particle diameter  $d_p = 100 \mu\text{m}$ , the filter efficiency decrease is not large and non linear. Finally, when the Reynolds number surpasses 35 000, the decrease in filter efficiency is rather low; 2–5%.

## GENERAL CONCLUSIONS

The air filtration process with fibrous filters media is recommended for laminar air flow but not for turbulent flow. The smaller the Reynolds number of infiltrated air, the highest filter media efficiency we obtain. When the average infiltrated particle diameter becomes smaller, the Reynolds number should be lower, in order to obtain a high efficiency of the filter media, due to the fact that as the particle diameter decreases, the filter efficiency decreases as well.

## REFERENCES

Koronakis, P. (2001) *Fluid mechanics*, 2nd ed., ION, Athens.

- Kowalski, W.J., Bahnfleth, W.P. & Whittam, T.S. (1999) Filtration of airborne microorganisms: Modeling and prediction. *ASHRAE Transactions*, 105, 4–17.
- Kundu, P.K. (1990) *Fluid mechanics*, Academic Press, San Diego.
- Lee, K.W. & Liu, B.Y.H. (1980) On the minimum efficiency and the most penetrating particle size for fibrous filters. *J. Air Pollution Control Association*, **30**(4), 377–381.
- Lienhard, J.H. V. & Lienhard, J.H. IV. (2003) *A heat transfer textbook*, 3rd ed., Phlogiston Press, Cambridge Massachusetts.
- Tien, C. (2012) *Principles of filtration*, 1st ed., Elsevier, Oxford.
- Yeh, H.C. & Liu, B.Y.H. (1974) Aerosol filtration by fibrous filters. *J. Aerosol Science*, **5**(2), 191–217.
- Wang, Q., Maze, B., Tafreshi, H.V. & Pourdeyhimi, B. (2006) Approaches for predicting collection efficiency of fibrous filters. *J. Textile and Apparel Technology and Management*, **5**(2), 1–7.