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A COMPARATIVE STUDY OF SOME OF THE SEDIMENT TRANSPORT EQUATIONS FOR AN ALLUVIAL CHANNEL WITH DUNES

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Abstract: The present work is a comparative evaluation of some of the well known sediment transport equations for the condition of dunes on the bed. It is fairly clear that no single equation provides reliable estimates of the total load of sediment transported for all types of bed forms. The most frequently occurring bed form being dunes, only this case is considered in this paper. The measurements of sediment transport were realized in the laboratory for various sediment sizes, utilizing a computerized tilting recirculation flume. The Yang equation (1973) was found to provide the best results for dunes.

Keywords: Sediment transport; recirculation flume; dune bed form; Yang equation

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INTRODUCTION

The sediment transport by the water courses are a source of many problems in the process of control and utilization of the surface waters. While the upstream erosion in a basin results in loss of soil and nutrients, the downstream deposition results in a loss of storage capacity of reservoirs, flooding etc. Since it is not possible to avoid these processes of nature, they can only be controlled reasonably when we have sufficient knowledge about the source and the quantity of sediment that is transported by these water courses.

The sediment transport process is quite complex and occurs in various modes, like, bed load, suspended load, wash load etc. The total load in transport at any given instant is the sum of the transportation in diverse modes. Hence, an accurate estimate of the total load depends on precise measurements of the combined bed load and suspension loads. Frequently, one of the various equations of transport available in the literature is utilized to obtain an estimate of the transport capacity or the sediment load being transported by a water course. However, the degree of confidence that can be placed on these results is not well established in the literature. One of the complicating factors is the occurrence of bed forms and different equations yield different results with each of the bed form that is present (Srinivasan & Curi, 1987), thus raising the basic question which of the equations can be used with confidence for a specific bed form? Habib (1993) has analyzed several of the equations of transport to estimate sediment discharge in rivers and concluded that all of them have drawbacks and proposes a new relationship for the sediment concentration in the flow. Yang & Lim (2003) propose yet another formula for total load based on dimensional analysis. It seems that a more rational approach to establish sediment transport relationships would be to associate them to specific bed forms for which they are best suited. The present paper is an attempt in that direction.

The occurrence of the bed forms is a process by which an equilibrium condition between the flow in the channel, its resistance and the sediment transport is established (Srinivasan & Hill, 1985). Ripples, dunes and flat bed are the most common forms of bed that occur and dunes are the predominant bed forms that occur in nature (Vanoni, 1975). A comparative performance of some well known transport equations for the condition of dunes is evaluated herein by means of precise laboratory measurements of the total amount of the sediment transported by the flow.

THE SEDIMENT TRANSPORT EQUATIONS

A large number of equations that attempt to estimate the total amount of sediment transported by rivers and channels are available in the literature. However, there are few publications that evaluate the performance of these equations under different conditions of transport (Nakato, 1990). Some equations like that of Einstein (1950) are based on the physical nature of the process of transport and others are empirical or semi-empirical. While some equations are meant only for the estimation of bed load or suspended load also exist equations that estimate the total load. However, most of these equations do not explicitly take into account the influence of the bed forms. In cases where the effect of the bed forms is considered it is considered only indirectly in which a correction is made to the total shear stress on the bed. The results of such corrections seem to be satisfactory only very occasionally.

Srinivasan & Curi (1987) compared the results obtained from eight well known equations of sediment transport utilizing laboratory channel data for the condition of plane bed. Their study showed that there exist very large variations in the calculated values when compared with the measured values. They found that the equation of Bagnold furnished the best results and the equation of Laursen would be even better when corrected for an observed phase difference. This study showed that there is not a single equation that may provide the most satisfactory results for all the bed forms and hence it may be wiser to identify the equation that provides the best results for each of the bed forms, separately.

In the present investigation, the dune bed form was selected for the comparative study as it is the most common bed form that occurs in nature. In choosing the equations that are frequently cited in the literature, the principal criteria utilized were (1) that they should be dimensionally homogeneous so that no conversions are needed from one system of measurement to the other, and (2) that all the types - empirical, semiempirical and theoretical - be represented in the evaluation. With these considerations, the equations selected were Meyer-Peter & Müller (1948), Einstein (1950), Laursen (1958), Bagnold (1966), Engelund & Hansen (1967), Yang (1973), Van Rijn (1984), and Zanke (1987) as presented in Dvwk (1990).

The basic data for the utilization of the above equations and the total amount of the transported sediments were obtained through experiments realized in recirculation channel at the Laboratory of Hydraulics of the Federal University of Campina Grande, Paraíba state, Brazil.

EXPERIMENTAL SETUP AND DATA COLLECTION

For a precise control of flow and accurate measurements of the basic data necessary for the study, a computerized adjustable slope recirculation channel was utilized. The system consist of a channel 16 m long, 0.50 m wide and 0.50 m deep with glass walls; a reservoir for the collection and pumping of water; a sediment collector;

Table 1. Experimental data obtained with dune bedform

Experiment number	Flow discharge (L/s)	Depth flow (m)	Water surface Slope (m/m)	Sediment discharge (g/s)
		Series 1: D_{50} = 0.35 mm		
1	28	0.11 657	0.00 310	22.94
2	28	0.12 040	0.00 340	28.81
3	28	0.14 933	0.00 136	9.10
4	32	0.16 907	0.00 147	2.08
5	36	0.15 117	0.00 350	21.74
6	36	0.17 197	0.00 113	8.06
7	36	0.15 597	0.00 217	13.53
8	32	0.14 483	0.00 319	9.15
9	24	0.12 143	0.00 276	23.59
10	20	0.10 263	0.00 244	7.92
11	16	0.08 487	0.00 250	9.17
12	40	0.22 087	0.00 112	1.71
		Series 2: D_{50} = 0.74 mm		
1	16	0.08 487	0.00 334	13.92
2	24	0.11 400	0.00 254	16.60
3	20	0.09 397	0.00 401	22.17
4	28	0.13 453	0.00 276	14.84
5	32	0.14 593	0.00 292	23.28
6	36	0.15 267	0.01 017	10.46
7	16	0.87 500	0.00 418	12.47
8	24	0.12 393	0.00 322	9.29
9	40	0.16 030	0.00 420	25.31
10	32	0.13 077	0.00 469	33.90
11	28	0.14 110	0.00 296	8.82

sediment and water pumps; and instruments for measurement and control. The flow is controlled by a computer through an inverter and a variable speed motor coupled to the pump by means of a continuous matching between the desired flow or hydrograph initially set and the actual flow discharge measured by an electro-magnetic flow meter.

The instrument carriage equipped with a bed profile probe and a velocity probe is controlled by another computer that makes it possible the collection of data at specific intervals of time in the same position or specific locations and intervals of distance along the length of the channel. A third computer linked to the control system stores and processes the experimental data. The experimental system was designed, fabricated and installed at Campina Grande, Brazil by a well known manufacturing Japanese firm from Kyoto.

The flow enters the channel at the upstream end from below and provisions are made to make the flow enter smoothly into the flume. The depth of flow is controlled by a stainless steel sectioned flap gate that permits the free passage of any transported sediment. Just downstream of the gate is located the sediment collection device which collects the total amount of sediment in the flow by filtering through the water. When samples are not being collected, the whole mixture of water and sediment falls into a hopper at the bottom of which the inlet of the sediment pump is located. The clear water filtered through the side screens of the hopper passes on to the external water tank in which the suction pipe of the water pump is located. Additional details of the setup can be found elsewhere (Srinivasan & Sawai, 1991).

COLLECTION OF DATA

With the experiments conducted, the main objective was to measure the total transport rate, the depth of flow and the water surface slope for a preset discharge rate and sediment size in the bed, maintaining dunes as the bed form. The viscosity of the water was obtained by measuring its temperature.

Two batches of sediments were selected for forming the erodible bed of the channel. The mean diameters were 0.35 and 0.74 mm, respectively. The sediments had a natural grading as they were obtained from conventional river sources. However, the extreme coarse and fine fractions were eliminated by sieving, thus maintaining essentially the sand fraction of the sediments. This resulted in a fairly uniform grain size distribution thereby avoiding the hiding effects in a heterogeneous mixture of sediments (Hsu & Holly, 1992).

The choice of the diverse discharges and depths of flow for the realization of the experiments was based on a preliminary set of experiments in which limiting conditions of depth and discharge were established to guarantee the occurrence of dunes during the main experiments. The preliminary runs were also helpful in getting a feeling on the sensitivity of the control gate for depth adjustment.

EXPERIMENTAL PROCEDURE

The experiments were realized in two series - one for each of the sediment sizes. After making a sediment bed of approximately 15 cm in the channel, an initial slope was arbitrarily set by tilting the channel. This procedure only quickened the time needed to set up equilibrium conditions and the real slope was measured from the water surface in the channel after a steady state had been established. A desired flow was chosen and set on the computer. The sediment pump and the water pump were switched on sequentially and after the set flow had been stabilized by the system. The control gate was adjusted such that a desired depth would result with the bed form being dunes. The equilibrium condition was verified by monitoring the depth of flow, bed pattern and the water surface slope in the central 10 m section of the flume avoiding any transitional effects at the ends.

Once the equilibrium conditions were secured, collection of sediment samples, measurement of the flow depth and the water surface slope were carried out. The depth of flow was established as the average difference between the water surface level and the deformed sediment bed level along the centre line of the channel measured at intervals of 20 cm. For each run three profiles were established for the bed and the water surface that resulted in an average depth for each profile. The depth of flow for the run was taken as the average of these three depths. In general, variations in depth from one profile to the other were very small but this procedure was adopted to avoid any errors in establishing the effective flow depth. In each run, eight samples of sediments were collected by filtering the flow through a collector during a fixed interval of time.

The sediment samples were dried in an oven and weighed. The average of the eight samples provided the sediment transport rate for each run. Totally, 23 runs were conducted with 12 runs in the first series with 0.35 mm sediment and 11 runs in the second series with the 0.74 mm sediment. The discharge ranged from 16 to 40 L/s while the depth of flow varied between 8.5 to 22.1 cm. The water surface slope ranged from 0.0012 to 0.01017 and solid transport varied from 2.08 to 33.71 g/s. The data for each of the runs is listed in **Table 1**.

RESULTS AND DISCUSSION

In order to calculate the transport rate by the selected equations it was necessary to calculate the bed hydraulic radius (r_b) and the procedure of Vanoni and Brooks (Vanoni, 1975) in which the total hydraulic radius is partitioned into the hydraulic radii of bed and side walls (side wall effect correction procedure) was utilized. Further, in the case of Einstein's equation, the bed hydraulic radius (r_b) was further subdivided into the hydraulic radius of bed grain (r_b') and the hydraulic radius of the bed forms (r_b'') according to the procedure suggested by Einstein and Barbarossa (Simon & Senturk, 1977). The Yang equation utilizes total bed hydraulic radius and other equations have their own method of correcting the bed shear stress. For each equation an algorithm was developed to calculate the total sediment transport rate and for each run and equation an index was established by dividing the calculated rate by the measured rate and this index was used as the basis for the evaluation of the chosen equations of transport.

For a comparative assessment of the different equations, the index ratios were grouped into six classes of variation with the lower and upper limits for each class being: 1/1.2 to 1.2 (class 1), 1/1.4 to 1.4 (class 2), 1/1.6 to 1.6 (class 3), 1/1.8 to 1.8 (class 4). 1/2.0 to 2.0 (class 5) and 1/3.0 to 3.0 (class 6). For each of the equations, the percentage of the index ratios in each of the classes was determined for all the 23 runs. **Table 2** shows the distribution of these percentages in each class for each of the equations.

It can be seen that the successive classes include all previous percentages cumulatively and for example class 5 includes all ratios between 50 and 200% that corresponds to all calculated values ranging from half the measured value up to twice the observed one. In principle, the equation that would present the highest value for any given class would be the best for that situation. Up to class 3 (1/1.6 to 1.6) no equation could be considered satisfactory as all of them resulted in a percentage of incidence of less than 60. Only for class 5 (1/2.0 to 2.0) a percentage of incidence above 80 could be found for at least one equation. Using this class as reference, the Yang equation presented the best results

Table 2. Distribution of the index ratios by class for each equation in percent

Equation Class	Müller	Hansen	Rijn	Yang	Zanke	Bagnold	Einstein	Laursen
1	8.70	8.70	8.70	21.75	13.05	4.35	0.00	0.00
2	17.40	17.40	21.75	43.50	26.10	8.70	0.00	0.00
3	17.40	26.10	30.45	56.55	39.15	14.40	0.00	0.00
4	21.75	34.80	43.50	69.60	47.85	30.45	0.00	0.00
5	26.10	34.80	43.50	82.65	47.85	43.50	0.00	0.00
6	52.20	43.50	73.95	82.65	65.25	69.60	8.70	34.80
All data	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00

with more than 82% of the observed ratios falling in this range. The second best - Zanke equation - presented only about 48% of the observed ratios in this range as can be seen from **Table 2**. The performance of the equations became better for class 6 (1/3.0 to 3.0) in which four equations - Rijn, Yang, Zanke and Bagnold - presented more than 65% of the index ratios in this class. The Einstein and Laursen equations proved to be the poorest of the lot. Teklie & Horlacher (2004) investigated the performance of some of the sediment transport equations utilizing the measured data in Kulfo River in Southern Ethiopia. They evaluated the transport equations by using the deviations of calculated sediment discharges from the measured data, which allowed them to rank the equations with respect to the measured data. They found that among the total load predictors, the equations of Bagnold, Engelund & Hansen and Yang were in good agreement with measured values. Assuming that the predominant bed form in Kolfu River observations was dunes, the results of the present study agrees well with their findings. It is interesting to know that though the two studies represent two different situations - a laboratory flume in one case and a natural river in another - the results are very consistent.

The studies of Srinivasan & Curi (1987) showed that the Bagnold equation and the adjusted Laursen equation to be best suited for obtaining sediment transport estimates for the case of the plane bed. The fact that these equations did not produce the same satisfactory results for the dunes seems to confirm the idea that no single equation may be best suited for estimating the sediment transport rates for all types of bed forms and it might be more practical to develop specific bed form based transport relationships to serve as predictors of total sediment transport in water courses.

CONCLUSIONS

The results of the present investigation show a clear superiority of the Yang (1973) equation for estimating the sediment transport in channel for the condition of dunes on the bed. The equations of Zanke and Van Rijn seem to be only moderately satisfactory. It may be necessary to evaluate the performance of the equations under a wider range of discharge and flow depths before arriving at a definitive conclusion. However, it seems very clear that no single equation would provide reliable estimates of the total load of sediment transport for all the bed forms that could sequentially or randomly occur in alluvial channels or natural water courses.

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