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COMPARISON OF EFFICIENCY OF A SOLAR DRIVEN CARNOT ENGINE UNDER MAXIMUM POWER AND POWER DENSITY CONDITIONS

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Abstract: A comparative analysis on thermodynamic efficiency based on maximum power & power density conditions have been performed for a solar-driven Carnot heat engine with internal irreversibility. In this analysis, the heat transfer from the hot reservoir is to be in the radiation mode and the heat transfer to the cold reservoir is to be in the convection mode. The thermodynamic efficiency function, power & power density functions have been derived and maximization of the power functions have been performed for various design parameters. From the optimum conditions, the thermal efficiencies at maximum power and power densities have been obtained. The effects of internal irreversibility, extreme temperature ratios & specific engine size in area ratio between the hot & cold reservoirs as various design parameters on thermodynamic efficiencies have been investigated for both the conditions. The efficiencies have been compared with Curzon-Ahlborn & Carnot efficiencies respectively. The analysis showed that the efficiency at maximum power output is greater than the efficiency at maximum power density. And the efficiencies can be greater than the Curzon-Ahlborn`s efficiency only for low values of design parameters.

Keywords: Power output, power density, Curzon-Ahlborn efficiency, internal irreversibility, design parameters

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INTRODUCTION

Solar driven heat engines using finite time heat transfer has been an important field of research. Power optimization studies of heat engines using finite time thermodynamics were started by Chambadal (1957) and Novikov (1957) and were continued by Curzon and Ahlborn (1975). Curzon and Ahlborn studied the performance in thermodynamic efficiency of an endoreversible Carnot heat engine at maximum power output. Using convective type linear heat transfer processes through finite temperature difference in both the hot and cold reservoirs, they obtained an upper limit to the endoreversible engine efficiency.

The study of irreversible thermodynamic cycles has been undertaken by many researchers after Curzon and Ahlborn's work. Sahin et al. (1995) studied the efficiency of a Joule-Brayton engine at maximum power density with consideration of engine size. Their results show that the efficiency at maximum power density is always greater than that presented by Curzon and Ahlborn (1975). Medina et al. (1996) extended the work of Sahin et al. (1995) to a regenerative Joule–Brayton cycle where the optimal operating conditions at the engine were expressed in terms of the compressor and turbine isentropic efficiencies and of the heat exchanger efficiency. The first finite time thermodynamic analysis for a solar driven heat engine was performed by Sahin[6]. He calculated the optimum operating temperature of the working fluid and the optimum efficiency of the solar-driven heat engine under maximum power output condition. Ozcaynak (1995) in his research works studied the effects of internal and external irreversibility parameters on thermal efficiency at maximum power conditions for an irreversible radiative solar driven heat engine model. Koyun (2004) carried out a comparative performance analysis based on maximum power and maximum power density criteria for a solar-driven heat engine with external irreversibilities. Sahin (2000) carried out optimization study based on maximum power criterion for an endoreversible solar driven heat engine by considering the collective role of radiation and convection heat transfer from hot reservoir & convection heat transfer to the cold reservoir. He also discussed the effects of the ratios of the reservoir temperatures and the heat transfer coefficients on the optimal performances. Sahin and Kodal (1995) applied this procedure to study the thermoeconomics of an endoreversible heat engine in terms of the maximization of a profit function defined as the quotient of the power output and the annual investment cost. Barranco-Jiménez et al. (2008) evaluated the optimum operation conditions of an endoreversible heat engine with different heat transfer laws when operated under maximum ecological function conditions. Further, Barranco-Jiménez et al. (2009) also applied thermo economic optimization and evaluated the optimum operation conditions of a solar-driven heat engine by considering three heads of performance viz. the maximum power, the maximum efficient power and the maximum ecological function. Barranco-Jiménez et al. (2011) studied the thermoeconomics of an irreversible solar driven heat engine by further considering losses due to heat transfer across finite time temperature differences (Zhou *et al.*, 2005; Chen *et al.*, 2004), heat leakage between thermal reservoirs (Wu et al.; Zheng et al.; Chen et al., 2008; Ladino-Luna, 2005; Ladino-Luna, 2007; Feidt, 2009) and the internal irreversibilities. In the present study, the thermodynamic efficiency at maximum power output is compared with the efficiency at maximum power density for various design parameters with consideration to internal irreversibilities. Further the efficiencies are compared with that of Curzon-Ahlborn`s efficiency and the conclusions are drawn.

MATERIALS AND METHODS

The present paper is on comparison of the thermodynamic efficiencies of solar driven Carnot heat engine based on maximum power and maximum power density considerations. The heat transfer from the hot reservoir as heat is taken in the inlet stroke of the engine is considered to be in the radiation mode and the heat transfer from the engine to the cold reservoir after producing power is considered to be in the convection mode. In addition, irreversibility effect is also considered during the heat transfers. In the next section, the basic theoretical modeling of the problem has been done and the resulting equations are derived. These equations are solved under the effect of various design parameters by writing suitable Matlab codes. The results obtained from the solution of the codes are analyzed and discussed in the results and discussion section, and finally the conclusions are drawn in the end.

THE THEORETICAL MODEL

The concept of an equilibrium and reversible Carnot cycle has played an important role in the development of classical thermodynamics. The reversible Carnot cycle has been used as an upper bound for heat engines. This upper bound of performance can only be achieved through a continuum of equilibrium states required for thermodynamically reversible processes, i.e. the equilibration rate for a change of state is infinitely faster than the rate of change of state. The T –S diagram of the considered reversible solar-driven heat engine with internal irreversibilities is shown in Fig. 1.

The heat engine operates between two extreme temperatures (TH and TL). If the heat transfer from the hot reservoir is assumed to be radiation dominated then the heat flow rate QH from the hot reservoir to the heat engine can be written as:

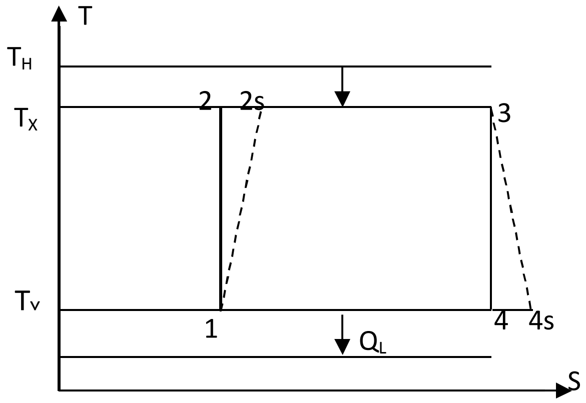

Fig. 1 T–S diagram of a reversible solar-driven heat engine with internal irreversibility.

$$\dot{Q}_H = C_H A_H (T_H{}^4 - T_X{}^4) \quad (1)$$

The heat flow rate Q_L from the heat engine to the cold reservoir, assuming convection dominance, can be written as:

$$\dot{Q}_L = C_L A_L (T_Y - T_L) \quad (2)$$

In **Eqs (1)** and **(2)**, CH and CL are the heat transfer coefficients of the hot and cold side heat exchangers, respectively. From the .first law of thermodynamics, the power output (W) of the cycle is:

$$W = Q_H - Q_L = C_H A_H (T_H{}^4 - T_X{}^4) - C_L A_L (T_Y - T_L) \quad (3)$$

The thermodynamic efficiency can be written as

$$\eta = 1 - \frac{\dot{Q}_L}{\dot{Q}_H} = 1 - \frac{C_L A_L (T_Y - T_L)}{C_H A_H (T_H{}^4 - T_X{}^4)} \quad (4)$$

Assuming an ideal gas, the maximum volume in the cycle V4 can be written as

$$V_4 = \frac{mRT_4}{P_4} \quad (5)$$

where m is the mass of the working fluid and R is the ideal gas constant. In the analysis, the minimum pressure in the cycle (P4) is taken to be constant. The power density, defined as the ratio of power to the maximum volume in the cycle [14–15] then takes the form

$$\dot{W}_d = \frac{\dot{W}}{V_4} = \frac{C_H A_H (T_H{}^4 - T_X{}^4) - C_L A_L (T_Y - T_L)}{\frac{mRT_4}{P_4}} \quad (6)$$

From the second law of thermodynamics for an irreversible cycle, the change in the entropies of the working fluid for heat addition and heat removing processes yields,

$$\int \frac{\partial \dot{Q}}{T} = \frac{\dot{Q}_H}{T_X} - \frac{\dot{Q}_L}{T_Y} < 0 \quad (7)$$

One can rewrite the inequality in Equation. (7) as

$$\frac{\dot{Q}_H}{T_X} = I \frac{\dot{Q}_L}{T_Y}, \ 0 < \mathrm{I} < 1 \quad (8)$$

With the above definition I becomes

$$I = \frac{\dot{Q}_H T_Y}{\dot{Q}_L T_X} = \frac{T_X (s_3 - s_2) T_Y}{T_Y (s_4 - s_1) T_X} = \frac{s_3 - s_2}{s_4 - s_1} \quad (9)$$

Substituting equation (1) & (2) in equation (6), we have

$$T_Y = \frac{T_L}{1 - \frac{C_H A_r (T_H{}^4 - T_X{}^4)}{I C_L T_X}} \quad (10)$$

where, $A_r = A_H/A_L$ = ratio of area of hot reservoir to cold reservoir, I = internal irreversibility. Substituting equation (9) in (3), (4) & (6), dimensionless power output ($\overline{W} = \frac{\dot{W}}{C_L A_L T_L}$), thermal efficiency & dimensionless power density output $\left(\overline{\mathrm{W}}_{\mathrm{d}} = \frac{\dot{\mathrm{W}}_{\mathrm{d}}}{\mathrm{C_L A_L} \, {}^{\mathrm{P_4}}\!/_{\mathrm{mR}}} \right)$ can be expressed as:

$$\eta = 1 - \frac{\tau}{\mathrm{I}\theta - \mathrm{A_r x}(1 - \theta^4)} \quad (11)$$

$$\overline{\mathrm{W}} = \frac{\mathrm{A_r} \eta \mathrm{x}(1 - \theta^4)}{\tau} \quad (12)$$

$$\overline{\mathrm{W}}_{\mathrm{d}} = \frac{\mathrm{A_r x}(1 - \theta^4)(\mathrm{I}\theta - \mathrm{A_r x}(1 - \theta^4) - \tau)}{\mathrm{I}\theta\tau} \quad (13)$$

where: $\theta = T_X / T_H$; τ = extreme temperature ratio $= T_L / T_H$; x = temperature constant $= C_H T_H^3 / C_L$. To Find the optimum working fluid temperature, θ under maximum power & power density conditions, **Eqs (11)** & **(12)** are differentiated with respect to θ and the resulting derivative is set to zero as:

$$4A_r^3 x^2\theta^{11} + 8A_r^2 I x \theta^8 - 8A_r^3 x^2 \theta^7 + 4A_r I^2 \theta^5 - (8A_r^2 I x + 3A_r I\tau)\theta^4 + 4A_r^3 x^2 \theta^3 - A_r I\tau = 0 \quad (13)$$

$$7A_r x \theta^8 + 4A_r I \theta^5 - (6x + 3\tau)A_r \theta^4 - A_r(\tau + x) = 0 \quad (14)$$

The optimum values of θ have to satisfy **Eqs (13)** and **(14)** for maximum power output and power density output conditions respectively. The solution of these equations can be done numerically. After finding the optimum θ values, the optimum efficiencies at maximum power and power density outputs respectively can be obtained by putting the values of θ in **Eq. (10)** under different designed conditions of extreme temperature ratio (τ), irreversibility factor (I) and specific engine size (A_r).

RESULTS AND DISCUSSION

The variations of thermodynamic efficiency at maximum power density output & power output conditions with specific engine size, A_r for different internal irreversibility parameter, I is shown in **Figs 2(a)** & **(b)** respectively which also includes the Carnot & Curzon- Ahlborn efficiencies for comparison. It is observed that the efficiency at maximum power density and maximum power output gradually decrease as the internal irreversibility increases(i,e,when I <<0.9) for any values of A_r. Both can only be greater than the Curzon-Ahlborn`s efficiency when internal irreversibility is as low as I = 0.9. Also as the specific engine size A_r increases, both the efficiencies decrease for all values of I. Further, one can observe from the figures that efficiencies at maximum power output (η_{mp}) is greater than the efficiencies at maximum power density (η_{mpd}) for all values of internal irreversibility parameter, I and specific engine size, A_r.

The variations of thermodynamic efficiency at maximum power density output & maximum power output conditions with respect to engine size, A_r for different extreme temperature ratios, τ are shown in **Fig. 3**. The Carnot & Curzon- Ahlborn efficiencies have also been incorporated for comparison. It is observed that the efficiency at maximum power density and maximum power output gradually decrease as τ increases for all A_r. Both efficiencies can only be greater than the Curzon- Ahlborn's efficiency when τ is as low as τ = 0.1. Also as the specific engine size A_r increases, both the efficiencies decrease for any values of τ. Further, as can be seen from the figure, efficiencies at maximum power output (η_{mp}) is greater than the efficiencies at maximum power density (η_{mpd}) for all values of extreme temperature ratios, τ and specific engine size, A_r.

The variations of thermodynamic efficiency at maximum power density output & power output conditions with extreme temperature ratio, τ for different values of specific engine size, A_r are shown in **Fig. 4(a)** & **(b)** respectively including the Carnot & Curzon- Ahlborn efficiencies. It is observed that the efficiencies at maximum power density and maximum power output conditions decrease with the increase in τ for all values of A_r. Further, similar to the findings in **Fig. 3**, efficiencies at maximum power output (η_{mp}) is greater than the efficiencies at maximum power density (η_{mpd}) for all values of extreme temperature ratios, τ and specific engine size, A_r.

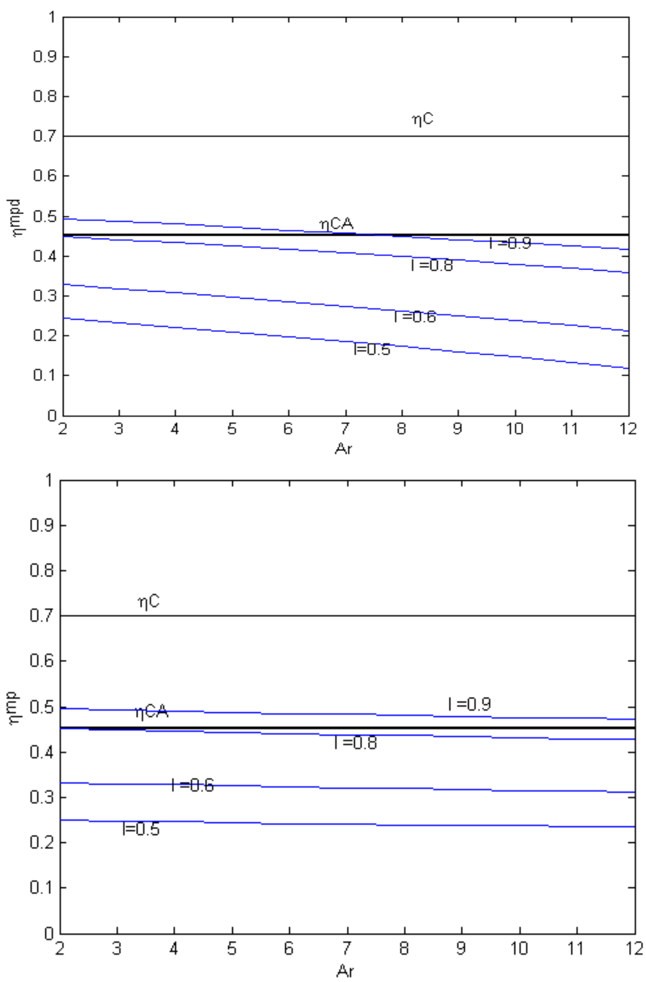

Fig. 2 (a) Variation of Carnot, Curzon-Ahlborn & Max power density efficiencies with A_r for different values of I (x= 0.01; τ= 0.3) (b). Variation of Carnot, Curzon-Ahlborn & Max power output efficiencies with A_r for different values of I (x= 0.01; τ= 0.3)

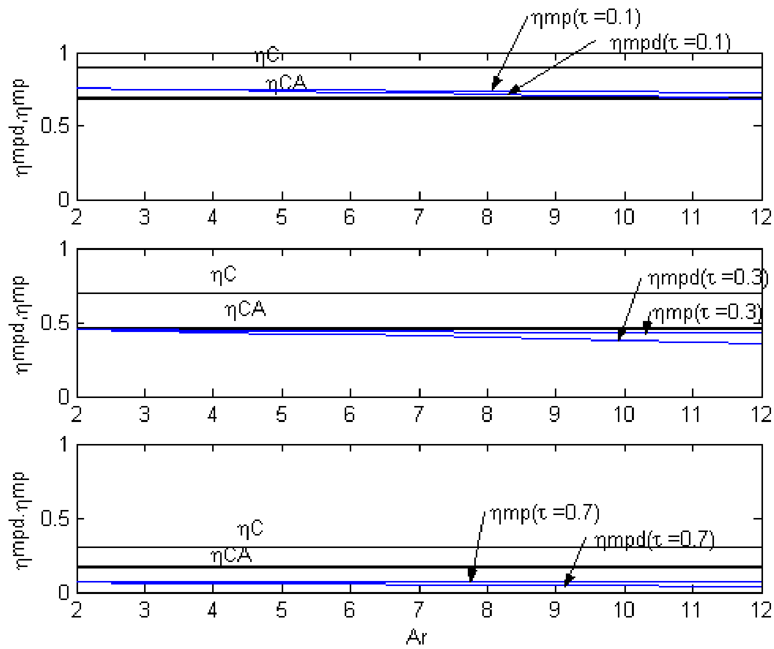

Fig 3. Variation of Carnot, Curzon-Ahlborn, Max. power & Max power density efficiencies with A_r for different values of τ (x= 0.01; I=0.8)

(a) (b)

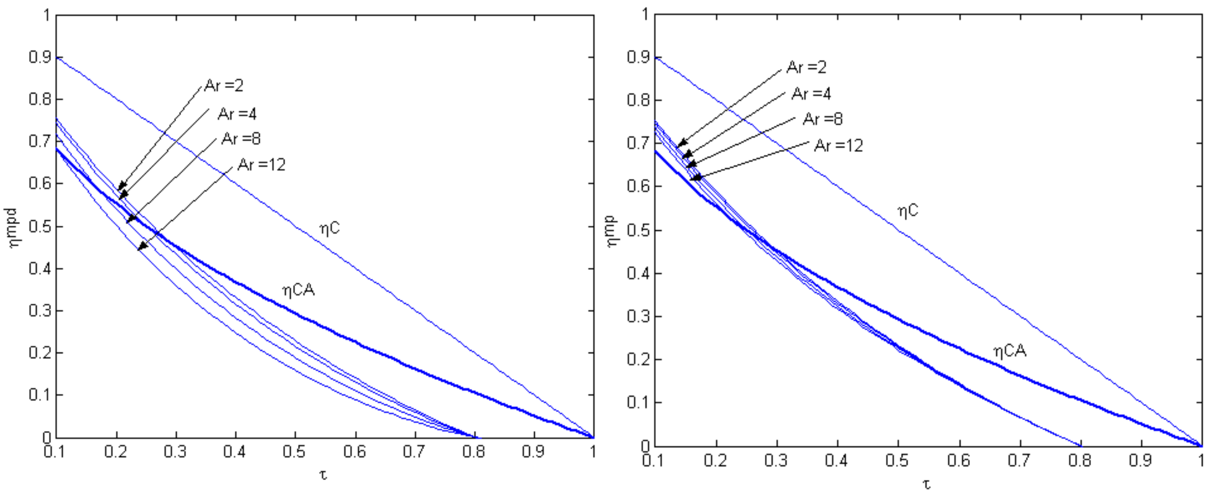

Fig. 4 (a) Variation of Carnot, Curzon-Ahlborn & Max power density efficiencies with extreme temperature ratio, [τ] for different values of A_r (I =0.8; x =0.01), (b). Variation of Carnot, Curzon-Ahlborn & Max power output efficiencies with extreme temperature ratio, [τ] for different values of A_r (I =0.8; x =0.01)

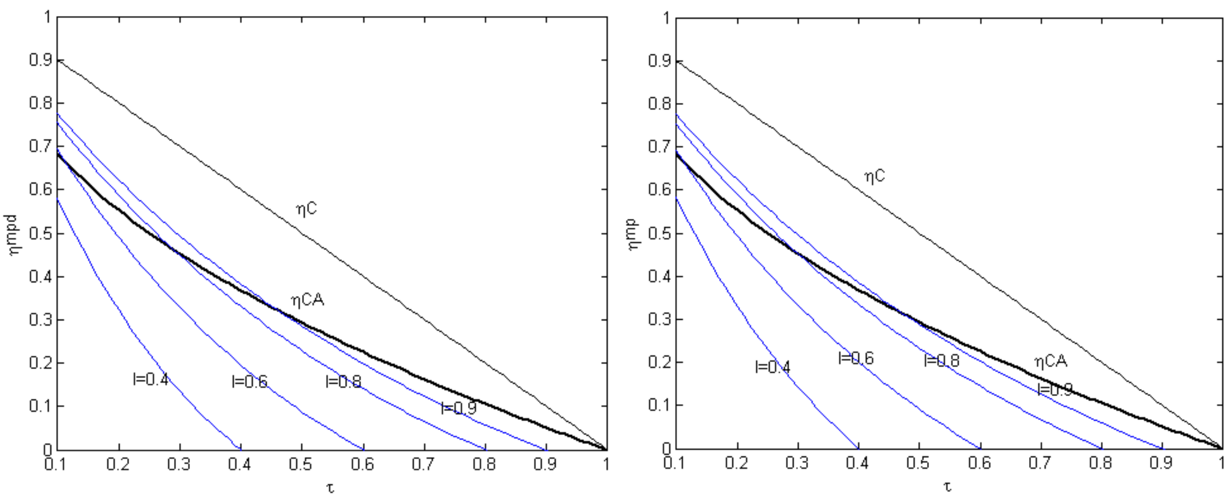

Fig. 5 (a). Variation of Carnot, Curzon-Ahlborn & Max power density efficiencies with extreme temperature ratio, [τ] for different values of I (A_r=2; x =0.01), (b). Variation of Carnot, Curzon-Ahlborn & Max power output efficiencies with extreme temperature ratio, [τ] for different values of I (A_r=2; x =0.01)

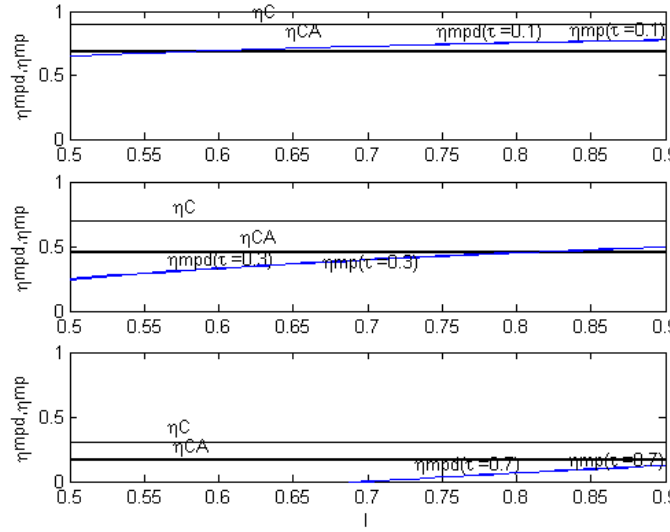

Fig. 6 Variation of Carnot, Curzon-Ahlborn, Max. power output & Max power density efficiencies with irreversibility, [I] for different values of τ (A_r=2; x =0.01)

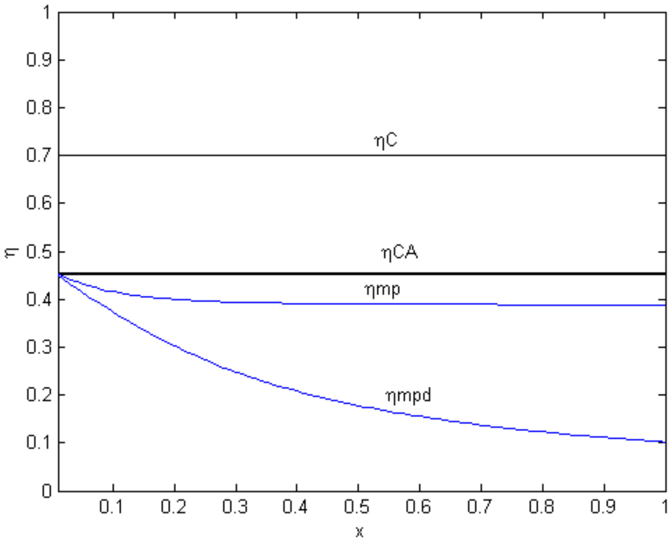

Fig. 8 Comparison of the dimensionless power and power density outputs with respect to temperature constant, x (τ =0.3; I= 0.8; A_r = 2).

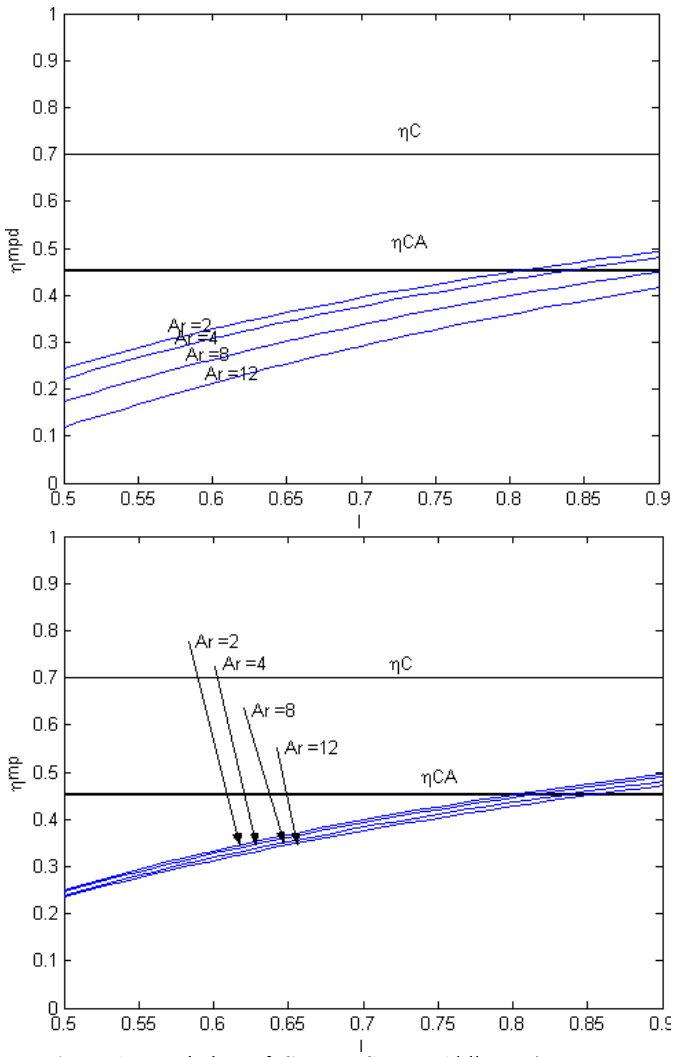

Fig. 7 (a) Variation of Carnot, Curzon-Ahlborn & Max power density efficiencies with irreversibility factor, [I] for different values of A_r (x = 0.01; τ = 0.3), (b). Variation of Carnot, Curzon-Ahlborn & Max power output efficiencies with irreversibility factor, [I] for different values of A_r (x = 0.01; τ = 0.3)

The variations of thermodynamic efficiency at maximum power density and power output conditions with the defined temperature constant, *x* is shown in **Fig. 8**. As can be seen from the figure, the efficiency at maximum power output is greater than the efficiency at maximum power density for the chosen values of the design parameters. And both of them are equal to the Curzon-Ahlborn's efficiency only when x is as low as 0.01. The typical values of *x* for solar driven heat engines are expected to be less than one[7].

CONCLUSION

In the present study, the thermodynamic efficiency at maximum power output is compared with the efficiency at maximum power density under the effect of various design parameters with consideration to internal irreversibilities. Two objective functions have been defined. One the ratio of power to the maximum volume in the cycle called power density and the other as simple power output function. These are optimized for various design parameters. And from the optimized conditions, the thermodynamic efficiencies have been evaluated at both power output and power density conditions. These efficiencies are compared with the Carnot and the Curzon-Ahlborn efficiencies under the similar conditions. The analysis showed that the efficiency at maximum power output is greater than the efficiency at maximum power density and that can be greater than the Curzon-Ahlborn`s efficiency only for low values of design parameters. The effect of internal irreversibility on optimal performance in efficiency has also been studied. The internal irreversibilities lead to reduction in thermodynamic efficiency for both conditions. This analysis may provide a basis for selection of design parameters for the optimal performance in thermodynamic efficiencies of a real solar powered heat engine.

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Nomenclature

C_H	hot side heat transfer coefficient
C_L	cold side heat transfer coefficient
$\dot{Q}$	rate of heat transfer
S	entropy
T	temperature
W	power output
W_d	power density output
η	efficiency
A_r	specific engine size
x	$= C_H T_H^3 / C_L$
τ	$= T_L / T_H$
θ	$= T_X / T_H$

Subscripts

C	Carnot
CA	Curzon–Ahlborn
H	heat source
L	heat sink
mp	maximum power condition
mpd	maximum power density condition

Superscript

-	dimensionless

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