# THE VALUE OF PRIVATE INFORMATION WITHIN EVOLVING GROUPS

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***Abstract***
*This note offers an analytical framework aimed at explaining how individual agents purposefully act with the goal of managing the value of their information sets. Agents undertake a process of private accumulation of information, which takes into account the non-rival nature of this peculiar entity. Non rivalry introduces an externality that might trigger long-term endogenous fluctuations. The dynamics of interaction, namely the possibility of entering or exiting the group to which the individuals belong, will determine time trajectories for the information flows that are unique for the specific conditions of interaction that are being considered at a given moment.*

***Keywords***: *Information flows. Group dynamics. Externalities. Nonrivalry. Endogenous fluctuations. Transactive memory.*

## O VALOR DA INFORMAÇÃO PRIVADA NO CONTEXTO DE GRUPOS EM EVOLUÇÃO

**Resumo**
Este artigo apresenta uma estrutura analítica que tem por objetivo explicar como é que os agentes individuais atuam, de modo intencional, com o propósito de gerir o valor da informação que detêm. Os agentes prosseguem um processo de acumulação privada de informação, o qual toma em consideração a natureza não rival desta entidade que detém características específicas. A não rivalidade introduz uma externalidade que pode despoletar flutuações endógenas de longo prazo. A dinâmica de interação, nomeadamente a possibilidade de entrar ou sair do grupo a que os indivíduos pertencem, vai determinar a formação de trajetórias no tempo para os fluxos de informação, as quais são únicas para as condições particulares de interação que estão a ser consideradas num determinado momento.

**Palavras chave**: Fluxos de informação. Dinâmica de grupo. Externalidades. Não rivalidade. Flutuações endógenas. Memória transactional.

## 1 INTRODUCTION

Information is an economic entity with special features. Its non rival and non exclusive character and its nature as a pervasive input in every economic activity, make it an important source of discussion in economics and managerial sciences. This letter introduces a simple model to explain the foundations on which the accumulation of information, by an individual agent on a given subject, is built upon. At each time period, some of the available information is lost, through obsolescence or depreciation, and new information is generated from a production function that has, as its single input, the information already obtained. Production of new information is subject to an externality effect, given the non rival nature that it involves. This externality is positive when the value of the information held by the whole of the group to which the individual currently belongs is larger than the value of the information that the individual possesses, and it is negative in the opposite circumstance. The intuition is that the individual benefits from being in contact with a group that owns relatively good information, but is penalized from interacting with a group that is in possession of information of poor quality (relatively to the one held by the agent herself).

The mechanism of generation of information is adapted from Gomes (2008), where a setup of competing technologies has been explored. In a similar way as in the mentioned paper, the externality effect will trigger a tension between information acquisition and information loss that, for particular conditions of the proposed environment, eventually generates endogenous fluctuations. The specific shape of the information flow will be determined basically by the number of individuals in the assumed group and by their initial endowments of information. An example will show that one cannot obtain the exact same trajectory for the value of an individual's information if one takes two different scenarios of interaction, no matter how close they are from each other (both in terms of initial conditions and of number of agents within the group).

The analysis to undertake may be applied in several contexts. For instance, it can be useful to approach, at a micro level, the re-shaping of social structures and the formation of groups given the profile of the individuals on the considered universe. This subject is explored, e.g., in Tantipathananandh, Berger-Wolf and Kempe (2007) and Tang, Wang and Liu (2011). It can also be a subject of interest for macroeconomics. It is by now well accepted that the economic science can no longer base most of its reasoning on the concept of representative agent. It is the interaction between heterogeneous agents within complex systems that will better describe real patterns of evolution of the economic variables. The economic system is a complex system characterized by agent heterogeneity, adaptation and evolution, out-of-equilibrium dynamics, path dependence and unique features that cannot be replicated by changing the dimension or the specificities of the interaction process. See Arthur, Durlauf and Lane (1997), Blume and Durlauf (2005), Markose (2005) and Holt, Rosser and Colander (2010) for an approach to economic complexity in systems of interacting agents.

The remainder of this note is organized as follows. Section 2 presents the model. Section 3 briefly addresses general dynamic properties. Section 4 highlights possible long-term outcomes by resorting to a numerical example. Section 5 concludes.

## 2 A DYNAMIC MODEL OF INFORMATION

Consider a discrete set of individuals $N \subset IN$, with *n* elements. Agent $i \in N$ is endowed, at a given time period, with a given amount of relevant information. The value of this information at time *t* is translated on the index $\omega_t^i$. The value of the information set will evolve according to the following dynamic rule,

$$\omega_{t+1}^i = (1-\delta)\omega_t^i + f(\omega_t^i)\xi^i\left(\omega_t^i, \frac{\sum_{j=1}^{n}\omega_t^j}{n}\right),\ \omega_0^i \text{ given} \quad (1)$$

Difference equation (1) describes a process through which, at time $t$+1, agent $i$ will hold the information that she already possessed at $t$, minus the lost information [$\delta\in(0,1)$ represents a rate of information obsolescence or depreciation] and plus the eventual generation of new information. New information emerges as characterized in the last term of the expression. One assumes a production function for information, $f(.)$, with private information (the one detained by the agent herself) being the only endogenous input. Function $f$ is supposed to be continuous and differentiable, and it will be subject to positive but decreasing marginal returns: $f'$>0 and $f''$<0. Below, it will be useful to assume an explicit functional form; let $f(\omega_t^i)=A(\omega_t^i)^\rho$, with $A$>0 a parameter that translates the contribution of other variables besides information for the generation of new information and $\rho\in(0,1)$ an elasticity parameter.

Function $\xi^i(.)$ respects to an externality that occurs in the process of creation of information. The externality intends to capture the following ideas:

1) If the average value of the information held by the whole of the individuals in a given group is larger than the value of information of agent $i$, this has a positive effect on the process of generation of new information for the individual agent, i.e., if

$$\frac{\sum_{j=1}^{n}\omega_t^j}{n}>\omega_t^i\text{, then }\xi^i>1;$$

2) If the average value of the information held by the whole of the individuals in a given group is lower than the value of information of agent $i$, this has a negative effect on the process of generation of new information for the individual agent, i.e., if

$$\frac{\sum_{j=1}^{n}\omega_t^j}{n}<\omega_t^i\text{, then }\xi^i<1;$$

3) If the average value of the information held by the whole of the individuals in a given group is exactly the same as the value of information of agent $i$, this has no effect on the process of generation of new information for the individual agent, i.e., if

$$\frac{\sum_{j=1}^{n}\omega_t^j}{n}=\omega_t^i\text{, then }\xi^i=1.$$

The above reasoning intends to describe a special feature of information as an economic good. It is harder to create new information when individuals already have an informational advantage relatively to their partners in the group relation. On the contrary, a positive externality is likely to arise when other individuals have more information, given the non rival nature of this entity.

A function that translates well the properties one has described above is the following,

$$\xi^i\left(\omega_t^i,\frac{\sum_{j=1}^{n}\omega_t^j}{n}\right)=1+\eta\arctan\left(\frac{\sum_{j=1}^{n}\omega_t^j}{n}-\omega_t^i\right)$$

Parameter $\eta$>0 measures how relevant the externality is for the generation of new information. When the informational advantage relatively to the other agents is large, the negative externality that emerges may imply a loss of the value of the information index relatively to the previous time period.

**3 PROPERTIES OF THE INFORMATION EQUATION**

Suppose that all agents in a group *N* are endowed with a same information technology, sharing the values of the parameters $\delta$, *A*, $\rho$ and $\eta$, and that the only thing that distinguishes them is initial endowments. In this case, it is straightforward to compute a steady-state value $\omega^*$ that will be common to all the individuals. This value is the one for which $\omega_{t+1}^i = \omega_t^i, \forall i \in N$. In this equilibrium point, externalities will be absent because all the individuals will share the same level of information. Thus, $\left(\xi^i\right)^* = 1$ and we compute $\omega^*$ by solving $\left(\omega^i\right)^* = (1-\delta)\left(\omega^i\right)^* + f\left[\left(\omega^i\right)^*\right]$. For the assumed *f* function, the steady-state level of information is $\left(\omega^i\right)^* = \left(A/\delta\right)^{1/(1-\rho)}$; this result is identical for every $i \in N$.

One can also address local dynamics. This is done by evaluating the behaviour of equation (1) in the vicinity of the steady-state relatively to an average agent. The average agent is the one for which the externality is always absent since she has no informational advantage or disadvantage relatively to all the other elements in the group. Let $\varpi_t^j$ be the value of the information stored by the average agent; the corresponding accumulation function is $\varpi_{t+1}^j = (1-\delta)\varpi_t^j + f\left(\varpi_t^j\right)$. By constructing the Jacobian matrix for the system of equations containing variables $\omega_t^i$ and $\varpi_t^j$ it becomes straightforward to find a condition ensuring stability. The linearized system in the vicinity of $\left(\omega^i\right)^*$ or $\left(\varpi^j\right)^*$ (they are identical) takes the form,

$$\begin{bmatrix} \omega_{t+1}^i - \left(\omega^i\right)^* \\ \varpi_{t+1}^j - \left(\varpi^j\right)^* \end{bmatrix} = \begin{bmatrix} 1-(1-\rho)\delta-(A/\delta^\rho)^{1/(1-\rho)} & (A/\delta^\rho)^{1/(1-\rho)} \\ 0 & 1-(1-\rho)\delta \end{bmatrix} \cdot \begin{bmatrix} \omega_t^i - \left(\omega^i\right)^* \\ \varpi_t^j - \left(\varpi^j\right)^* \end{bmatrix}$$

The eigenvalues of the Jacobian matrix are $\lambda_1 = 1-(1-\rho)\delta-(A/\delta^\rho)^{1/(1-\rho)}$ and $\lambda_2 = 1-(1-\rho)\delta$. Eigenvalue $\lambda_2$ falls inside the unit circle, for any admissible parameter values, and thus one can guarantee the existence of at least one stable dimension. Full stability will require $\lambda_1$ to be larger than -1 (observe that it is necessarily lower than 1), i.e., stability holds under condition $1-(1-\rho)\delta-(A/\delta^\rho)^{1/(1-\rho)} > -1$. The condition can be rearranged and presented as an upper bound on parameter *A*; the inequality comes $A < \delta^\rho\left[2-(1-\rho)\delta\right]^{1-\rho}$. If this condition does not hold, saddle-path stability will characterize the dynamics of the system in the steady-state vicinity; this result hides, as one will confirm in the next section, the possibility of regular or irregular bounded instability, what is revealed when looking at the global dynamic properties of the process one is characterizing.

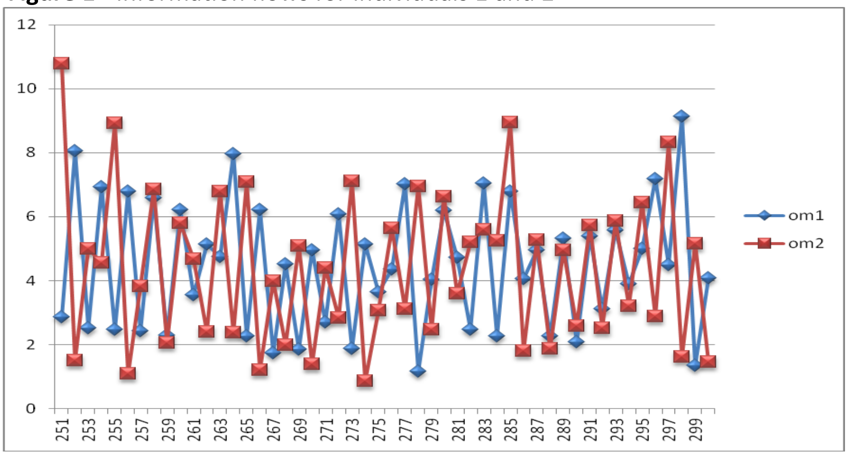
**4 NUMERICAL ILLUSTRATION**

As previously mentioned, a group of *n* individuals is taken into account. They share the same parameter values in their information dynamic equation and they differ solely with respect to one feature: the initial endowment of information. To make a concrete analysis of information flows within the group, let us consider a numerical example. Start by assuming the set of parameter values ($\delta$,*A*,$\rho$,$\eta$)=(0.05,0.6,0.75,2.5) and take *n*=5. The selected initial endowments are the following: $\omega_0^1 = 12$, $\omega_0^2 = 3$, $\omega_0^3 = 7$, $\omega_0^4 = 6$, $\omega_0^5 = 1$. The steady-state value of information, common to all the individuals in the group, is straightforward to compute: $\left(\omega^i\right)^* = \left(0.6/0.05\right)^{1/(1-0.75)} = 20736$. However, there will be convergence to this equilibrium value only if the stability condition presented in the previous section holds; this does not occur for the selected parameter values.[1]

The absence of stability will imply some other kind of long-term outcome. Specifically, for the initial values one has assumed, we identify the presence of everlasting irregular cycles. These endogenous fluctuations maintain the values of the information variables close to their initial levels - the process of information creation and information destruction, through interaction on the accumulation of a non rival good, implies this result.

Figure 1 displays the motion of the information flow for two series, the ones respecting to the first two individuals in the group. The transient phase is excluded; the presented observations go from *t*=251 to *t*=300. It is visible, in the figure, a pattern of evolution of the information variables characterized by irregular fluctuations, which are exclusively determined by the rule of interaction one has assumed.

**Figure 1 -** Information flows for individuals 1 and 2

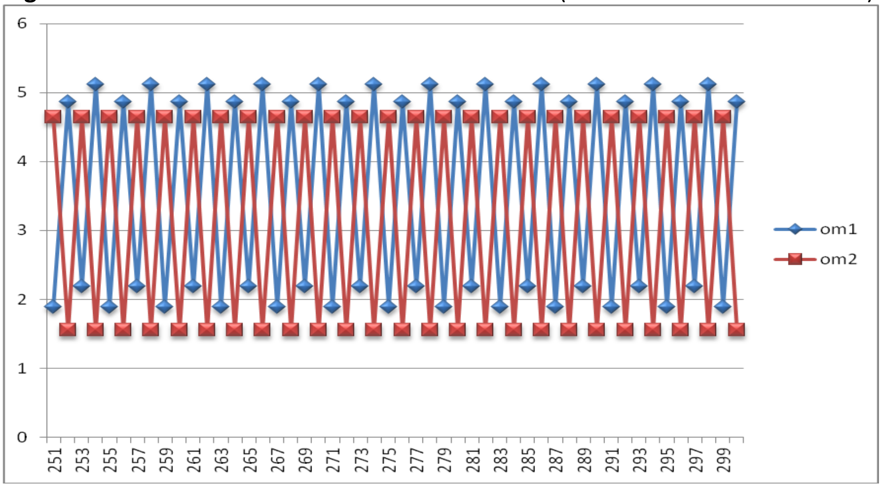



Source: the authors

The evolution of $\omega_t^1$ and $\omega_t^2$ in figure 1 can only be found under the exact conditions in which the interaction process was established. If one changes a single feature of the environment, this has consequences over the whole system, implying differently shaped information flows. Consider, for instance, that the initial endowment of information of agent 4 is $\omega_0^4 = 3$, instead of the value that was previously taken. This will change information trajectories in time for all the individuals in the group. In this particular case, for the same agents for which figure 1 was drawn, we no longer find evidence of irregular fluctuations. The system will rest in a state such that $\omega_t^1$ follows a period 4 cycle while $\omega_t^2$ will be locked in a cycle of periodicity 2. This is displayed in Figure 2.

[1] For the values of $\delta$ and $\rho$ that were given, the required constraint on *A*, for stability, would be *A*<0.1255

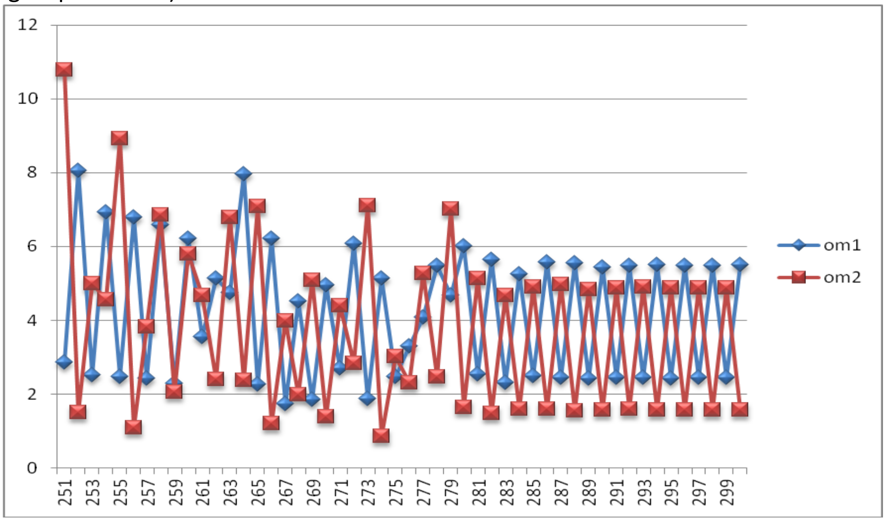
**Figure 2 -** Information flows for individuals 1 and 2 (with a modified initial value)




Source: the authors

The example has illustrated that initial conditions matter for the dynamics of information flows. For a given set of initial values, one might also cause a change in the structure of interaction by introducing elements into the group or by removing elements from the group. As a further illustration, recover the original set of initial values and consider that at *t*=275, the fourth and the fifth individuals abandon the group. The pattern of interaction necessarily changes. In this specific case, the irregular path initially observed gives place to a period 2 cycle for both individuals 1 and 2, as one can observe in Figure 3.

**Figure 3 -** Information flows for individuals 1 and 2 (when individuals 4 and 5 abandon the group at *t*=275)




Source: the authors

Even though no change was introduced in the structure of the model (in the underlying parameter values or in the shape of the functions the dynamic rule contains), one observes that the interaction is sensitive to initial endowments and to the number of individuals in the group. This illustrates the complexity of the dynamics of a group, when a connection is established between the respective decision processes.

## 5 CONCLUSION

Individuals accumulate information on a given subject resorting to their own previous knowledge on the environment that surrounds them. However, information has a non rival nature and, as a consequence, the interaction with others is essential, as well, in terms of generation / destruction of the information that the individual agent possesses. Therefore, to understand the dynamics of information flows, one needs to analyze how the group under analysis is effectively organized and structured. A numerical inspection has allowed to conclude that even when the individual agent maintains intact her own capabilities to accumulate information, slight changes on the behavior of others or of the system itself (e.g., the entering or exiting of new members on the group) may induce a radical modification\ on the qualitative nature of the corresponding information flow. For instance, completely erratic time trajectories can be transformed in period 2 cycles just by changing the composition of the group.

The analysis has allowed to illustrate how individual conditions might suffer a relevant impact from changes occurring within the considered group. The setup can be adapted to different environments, but it is specially suited to deal with issues of transactive memory (i.e., the memory generated and stored within group relations), namely in what concerns applications in the study of human resources management.

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